UNIT-I

SIGNALS & SYSTEMS

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Signal : A signal is defined as a time varying physical phenomenon which is intended to convey information. (or) Signal is a function of time. (or) Signal is a function of one or more independent variables, which contain some information.

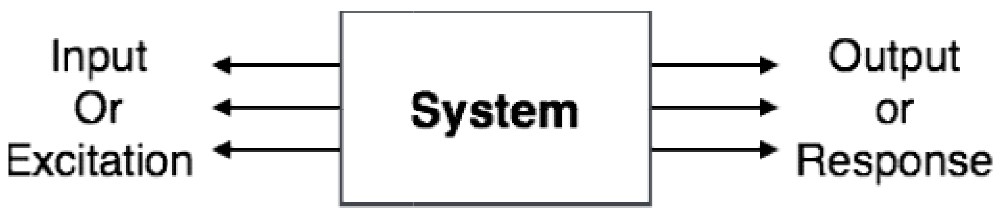
Example: voice signal, video signal, signals on telephone wires , EEG, ECG etc.

Signals may be of continuous time or discrete time signals.

System : System is a device or combination of devices, which can operate on signals and produces corresponding response. Input to a system is called as excitation and output from it is called as response.

For one or more inputs, the system can have one or more outputs.

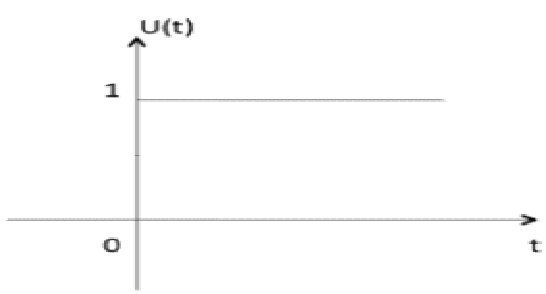
Example: Communication System



Elementary Signals or Basic Signals:

Unit Step Function

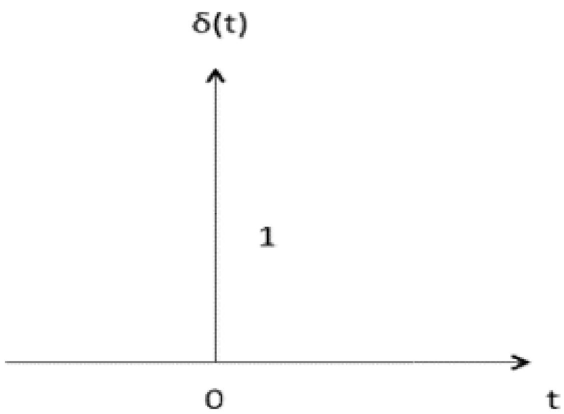
Unit step function is denoted by u(t). It is defined as u(t) = 1 when $t \geq 0$ and
0 when $t < 0$



- It is used as best test signal.
- Area under unit step function is unity.

Unit Impulse Function

Impulse function is denoted by δ(t). and it is defined as δ(t) = $\begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$

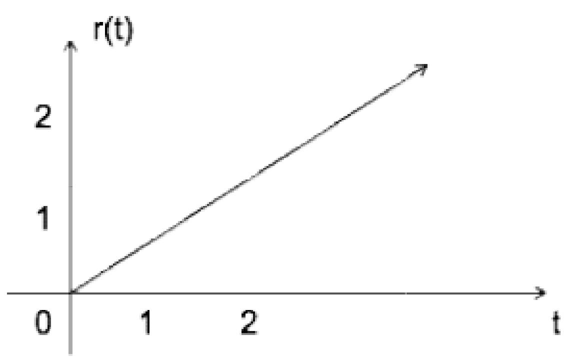


$$\int_{-\infty}^{\infty} \delta(t) dt = u(t)$$

$$\delta(t) = \frac{du(t)}{dt}$$

Ramp Signal

Ramp signal is denoted by r(t), and it is defined as r(t) = $\begin{cases} t & t \geqslant 0 \\ 0 & t < 0 \end{cases}$



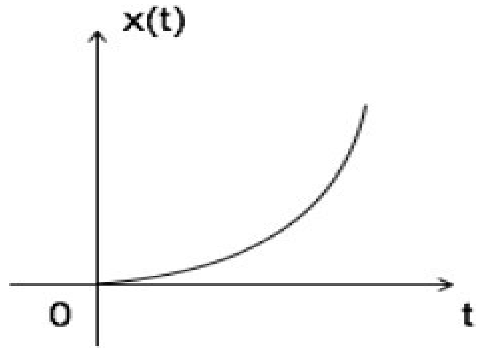
$$\int u(t) = \int 1 = t = r(t)$$

$$u(t) = \frac{dr(t)}{dt}$$

Area under unit ramp is unity.

Parabolic Signal

Parabolic signal can be defined as x(t) = $\begin{cases} t^2/2 & t \geqslant 0 \\ 0 & t < 0 \end{cases}$



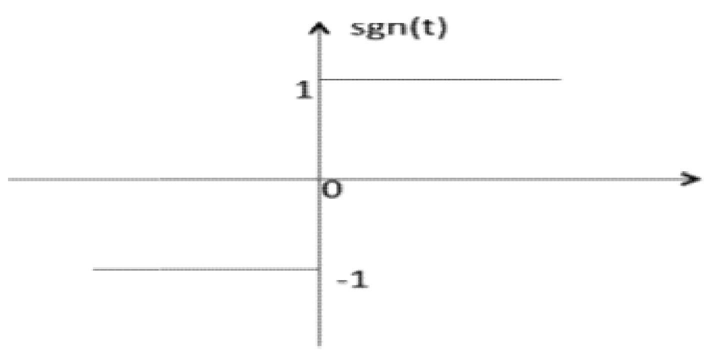
$$\iint u(t)dt = \int r(t)dt = \int t dt = \frac{t^2}{2} = parabolic signal$$

$$\Rightarrow u(t) = \frac{d^2x(t)}{dt^2}$$

$$\Rightarrow r(t) = \frac{dx(t)}{dt}$$

Signum Function

Signum function is denoted as sgn(t). It is defined as sgn(t) = $\begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$



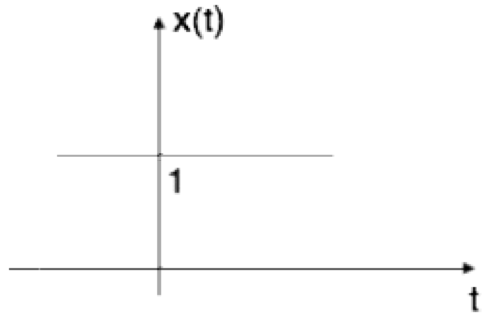
sgn(t) = 2u(t) – 1

Exponential Signal

Exponential signal is in the form of x(t) = $e^{\alpha t}$

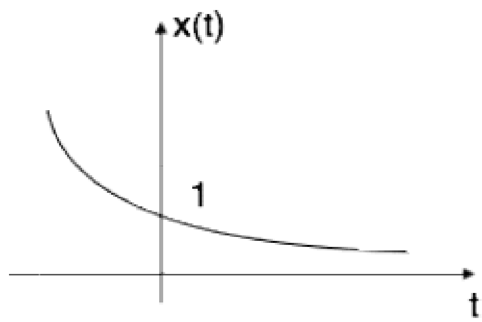
The shape of exponential can be defined by α

Case i: if $\alpha = 0 \longrightarrow$ x(t) = e^0= 1



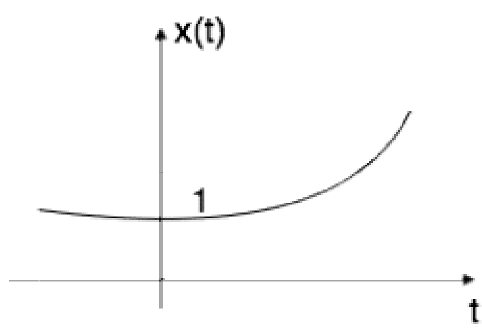
Case ii: if α< 0 i.e. -ve then x(t) = $e^{-\alpha t}$

. The shape is called decaying exponential.



Case iii: if α> 0 i.e. +ve then x(t) = $e^{\alpha t}$

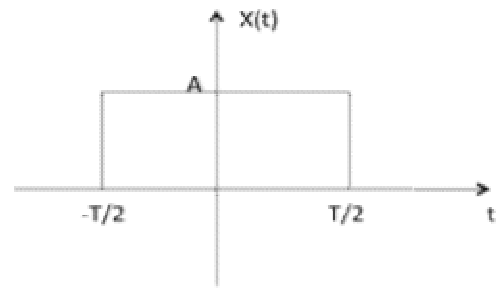
. The shape is called raising exponential.

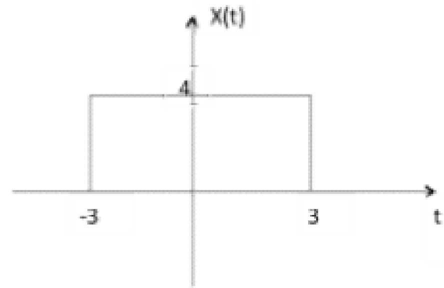


Rectangular Signal

Let it be denoted as x(t) and it is defined as

$$x(t) = A\ rect\left[\frac{r}{T}\right] \qquad \text{ex: } 4\ rect\left[\frac{r}{6}\right]$$

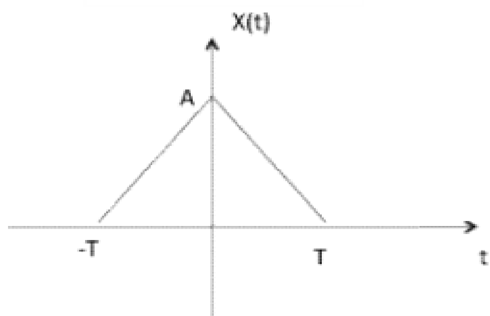


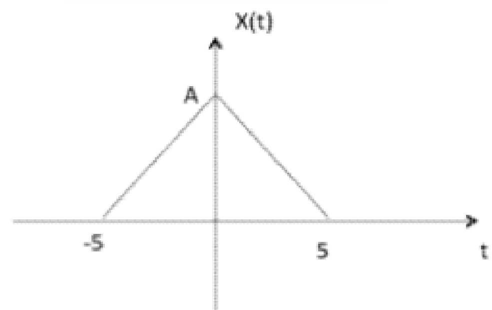


Triangular Signal

Let it be denoted as x(t)

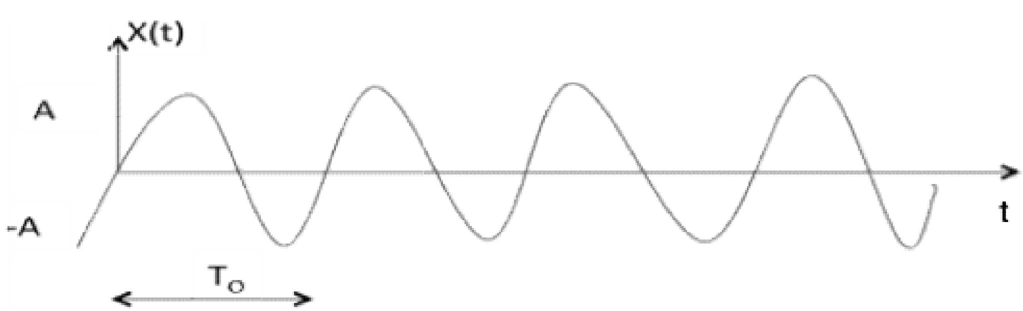
$$x(t) = A\left[1 - \frac{|t|}{T}\right] \qquad \text{ex: } x(t) = A\left[1 - \frac{|t|}{5}\right]$$





Sinusoidal Signal

Sinusoidal signal is in the form of x(t) = A cos($w_0 \pm \phi$) or A sin($w_0 \pm \phi$)



Where $T_0 = 2\pi/w_0$

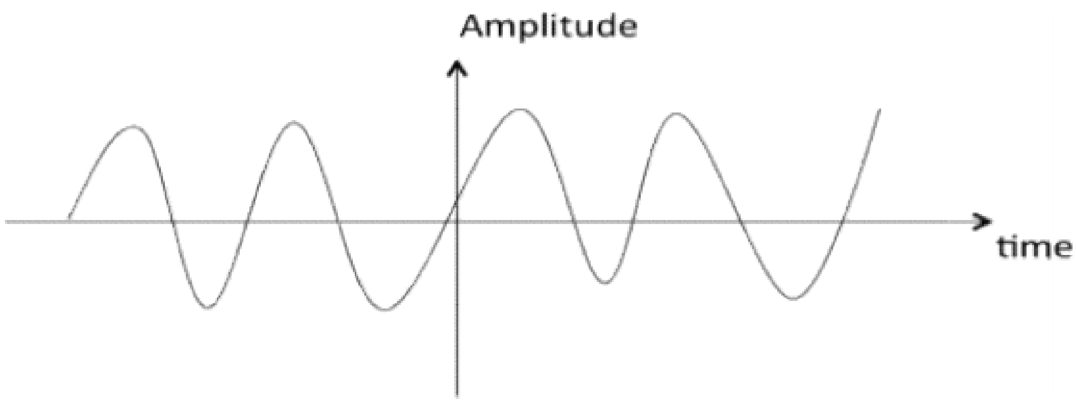
Classification of Signals:

Signals are classified into the following categories:

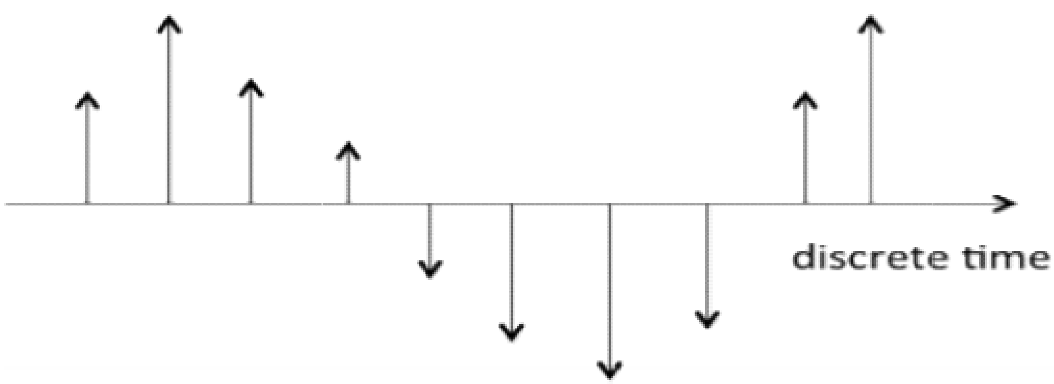
- Continuous Time and Discrete Time Signals
- Deterministic and Non-deterministic Signals
- Even and Odd Signals
- Periodic and Aperiodic Signals
- Energy and Power Signals
- Real and Imaginary Signals

Continuous Time and Discrete Time Signals

A signal is said to be continuous when it is defined for all instants of time.

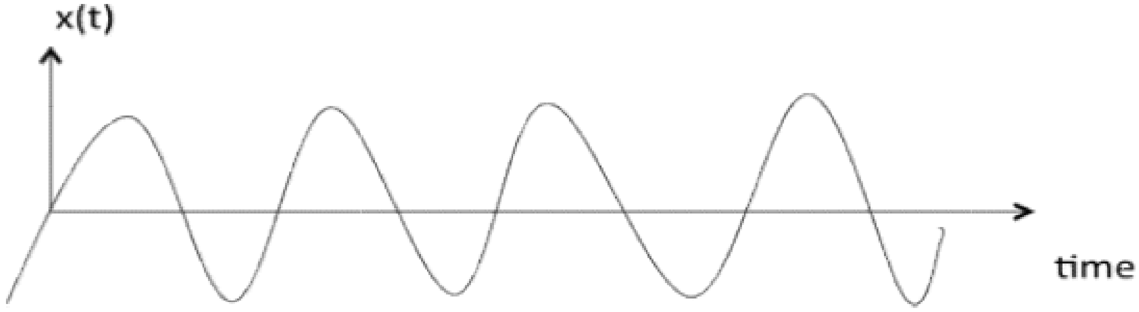


A signal is said to be discrete when it is defined at only discrete instants of time/

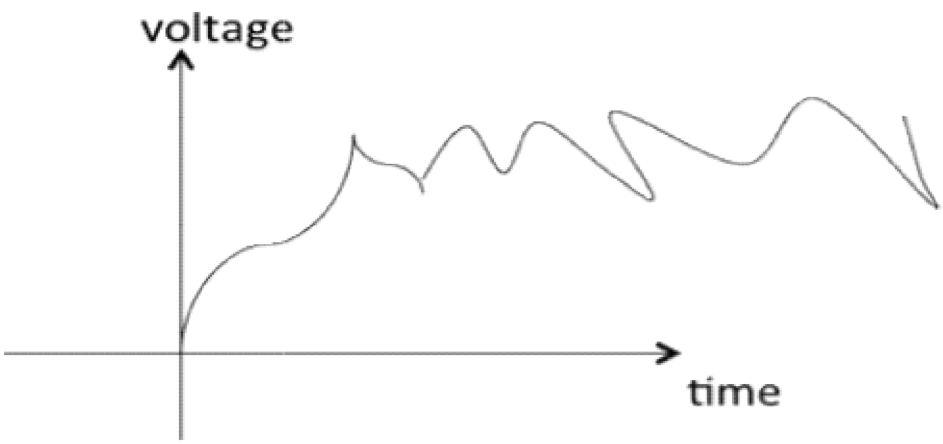


Deterministic and Non-deterministic Signals

A signal is said to be deterministic if there is no uncertainty with respect to its value at any instant of time. Or, signals which can be defined exactly by a mathematical formula are known as deterministic signals.



A signal is said to be non-deterministic if there is uncertainty with respect to its value at some instant of time. Non-deterministic signals are random in nature hence they are called random signals. Random signals cannot be described by a mathematical equation. They are modelled in probabilistic terms.



Even and Odd Signals

A signal is said to be even when it satisfies the condition $x(t) = x(-t)$

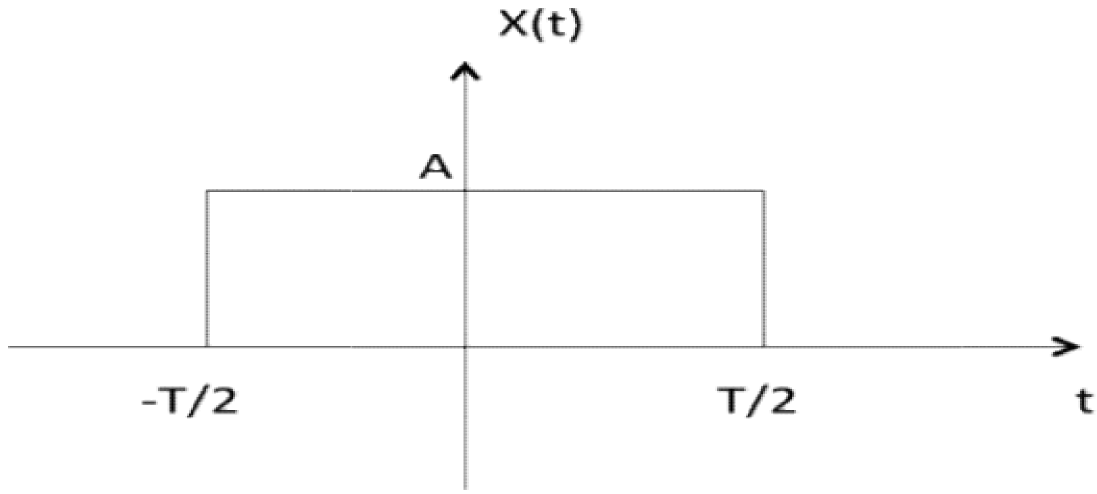
Example 1: t^2, t^4… cost etc.

Let $x(t) = t^2$

$x(-t) = (-t)^2 = t^2 = x(t)$

$\therefore t^2$ is even function

Example 2: As shown in the following diagram, rectangle function $x(t) = x(-t)$ so it is also even function.



A signal is said to be odd when it satisfies the condition x(t) = -x(-t)

Example: t, t^3 ... And sin t

Let x(t) = sin t

x(-t) = sin(-t) = -sin t = -x(t)

∴ sin t is odd function.

Any function f(t) can be expressed as the sum of its even function f_e(t) and odd function f_o(t).

$f(t) = f_e(t) + f_0(t)$

where

$f_e(t) = ½[f(t) + f(-t)]$

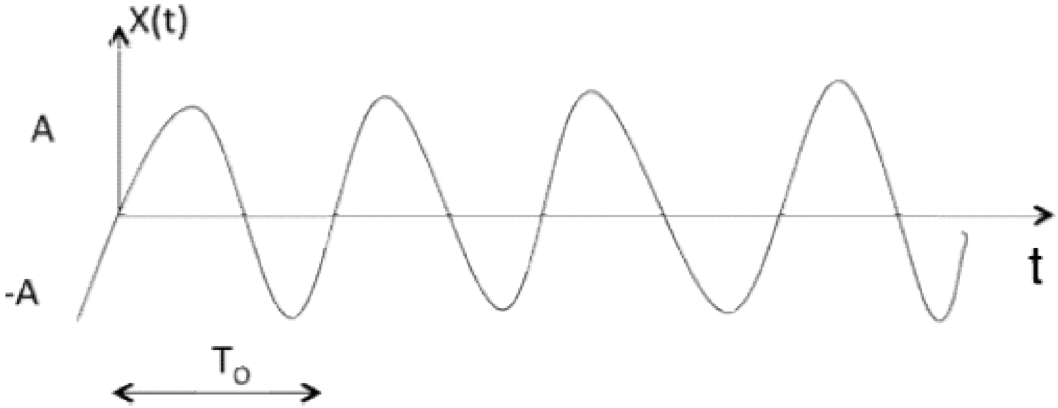
Periodic and Aperiodic Signals

A signal is said to be periodic if it satisfies the condition x(t) = x(t + T) or x(n) = x(n + N).

Where

T = fundamental time period,

1/T = f = fundamental frequency.



The above signal will repeat for every time interval T_0 hence it is periodic with period T_0.

Energy and Power Signals

A signal is said to be energy signal when it has finite energy.

$$\text{Energy } E = \int_{-\infty}^{\infty} x^2\,(t)dt$$

A signal is said to be power signal when it has finite power.

$$\text{Power } P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)dt$$

NOTE:A signal cannot be both, energy and power simultaneously. Also, a signal may be neither energy nor power signal.

Power of energy signal = 0
Energy of power signal = ∞

Real and Imaginary Signals

A signal is said to be real when it satisfies the condition x(t) = x*(t)
A signal is said to be odd when it satisfies the condition x(t) = -x*(t)
Example:

If x(t)= 3 then x*(t)=3*=3 here x(t) is a real signal.
If x(t)= 3j then x*(t)=3j* = -3j = -x(t) hence x(t) is a odd signal.

Note: For a real signal, imaginary part should be zero. Similarly for an imaginary signal, real part should be zero.

Basic operations on Signals:

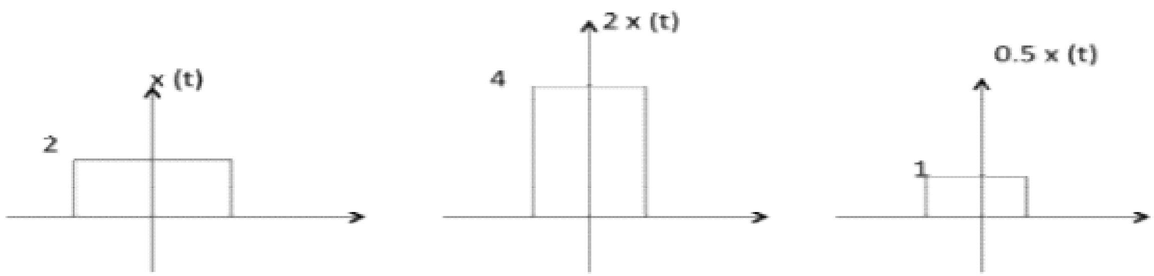
There are two variable parameters in general:

1. Amplitude
2. Time

(1) The following operation can be performed with amplitude:

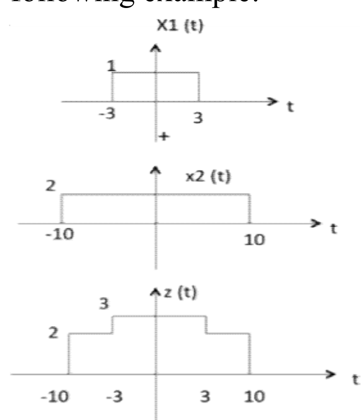
Amplitude Scaling

C x(t) is a amplitude scaled version of x(t) whose amplitude is scaled by a factor C.



Addition

Addition of two signals is nothing but addition of their corresponding amplitudes. This can be best explained by using the following example:



As seen from the previous diagram,

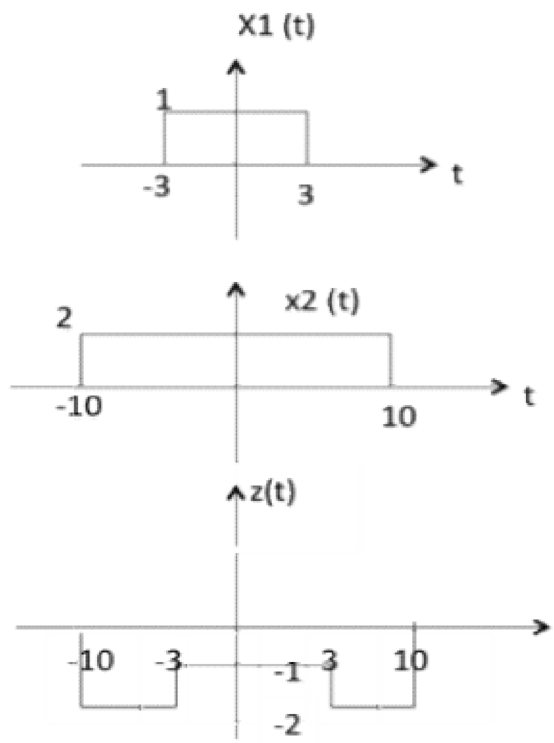
$$-10 < t < -3 \text{ amplitude of } z(t) = x_1(t) + x_2(t) = 0 + 2 = 2$$

$$-3 < t < 3 \text{ amplitude of } z(t) = x_1(t) + x_2(t) = 1 + 2 = 3$$

$$3 < t < 10 \text{ amplitude of } z(t) = x_1(t) + x_2(t) = 0 + 2 = 2$$

Subtraction

subtraction of two signals is nothing but subtraction of their corresponding amplitudes. This can be best explained by the following example:



As seen from the diagram above,

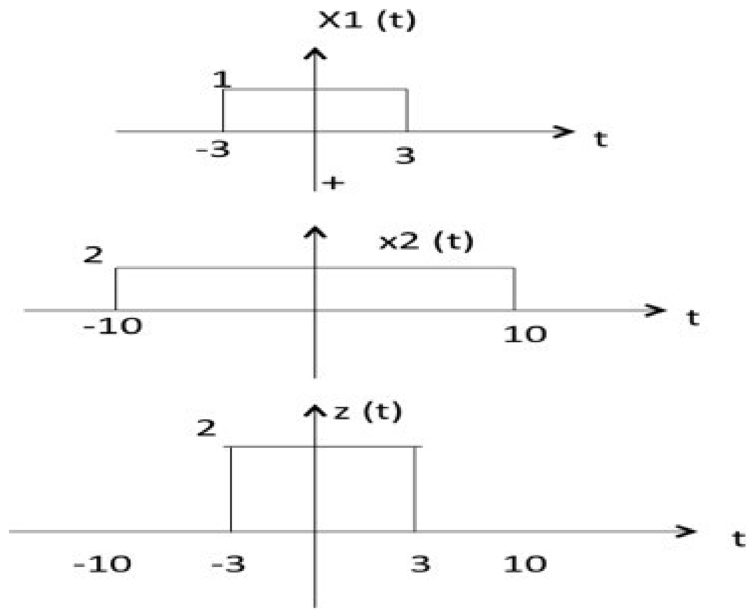
-10 < t < -3 amplitude of z (t) = $x_1(t) - x_2(t) = 0 - 2 = -2$

-3 < t < 3 amplitude of z (t) = $x_1(t) - x_2(t) = 1 - 2 = -1$

3 < t < 10 amplitude of z (t) = $x_1(t) - x_2(t) = 0 - 2 = -2$

Multiplication

Multiplication of two signals is nothing but multiplication of their corresponding amplitudes. This can be best explained by the following example:



As seen from the diagram above,

$-10 < t < -3$ amplitude of z (t) = $x_1(t) \times x_2(t) = 0 \times 2 = 0$
$-3 < t < 3$ amplitude of z (t) = $x_1(t) - x_2(t) = 1 \times 2 = 2$
$3 < t < 10$ amplitude of z (t) = $x_1(t) - x_2(t) = 0 \times 2 = 0$

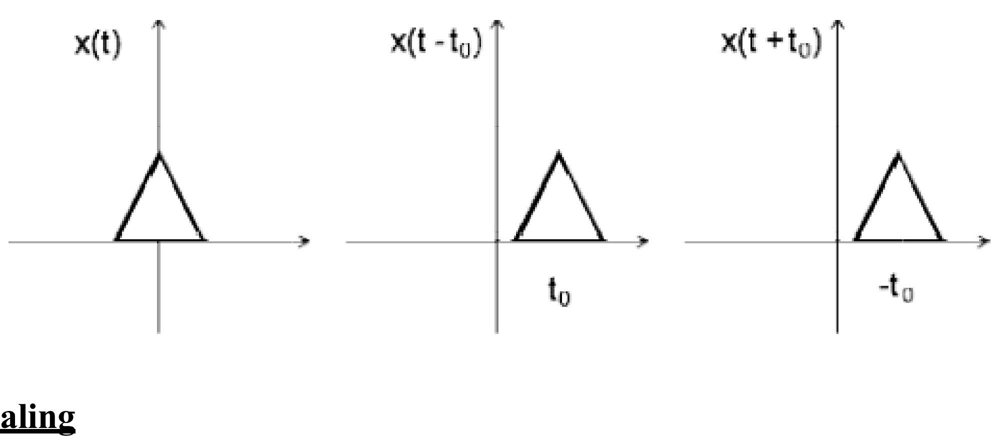
(2) The following operations can be performed with time:

<u>Time Shifting</u>

$x(t \pm t_0)$ is time shifted version of the signal x(t).

$x\ (t + t_0) \longrightarrow$negative shift

$x\ (t - t_0) \longrightarrow$positive shift

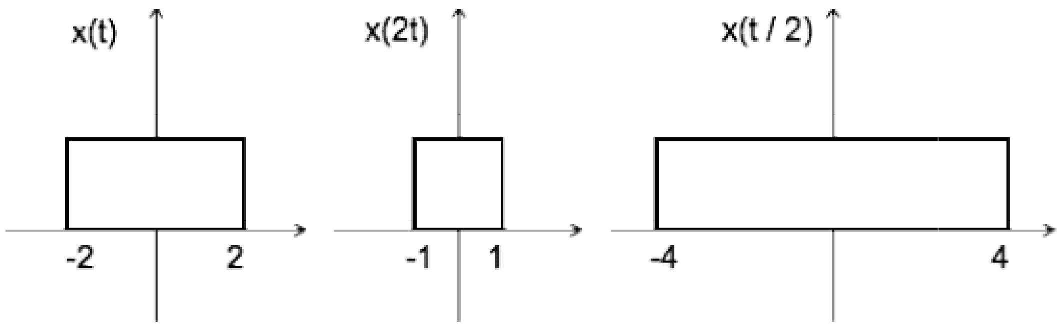


<u>Time Scaling</u>

x(At) is time scaled version of the signal x(t). where A is always positive.

|A| > 1 ⟶ Compression of the signal

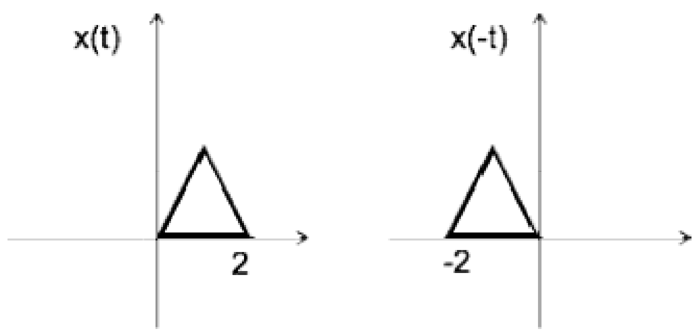
|A| < 1 ⟶ Expansion of the signal



Note: u(at) = u(t) time scaling is not applicable for unit step function.

Time Reversal

x(-t) is the time reversal of the signal x(t).



Classification of Systems:

Systems are classified into the following categories:

- Liner and Non-liner Systems
- Time Variant and Time Invariant Systems
- Liner Time variant and Liner Time invariant systems
- Static and Dynamic Systems
- Causal and Non-causal Systems
- Invertible and Non-Invertible Systems
- Stable and Unstable Systems

Linear and Non-linear Systems

A system is said to be linear when it satisfies superposition and homogenate principles. Consider two systems with inputs as $x_1(t)$, $x_2(t)$, and outputs as $y_1(t)$, $y_2(t)$ respectively. Then, according to the superposition and homogenate principles,

$$T [a_1 x_1(t) + a_2 x_2(t)] = a_1 T[x_1(t)] + a_2 T[x_2(t)]$$

$$\therefore T [a_1 x_1(t) + a_2 x_2(t)] = a_1 y_1(t) + a_2 y_2(t)$$

From the above expression, is clear that response of overall system is equal to response of individual system.

Example:

$y(t) = x^2(t)$

Solution:

$y_1 (t) = T[x_1(t)] = x_1^2(t)$

$y_2 (t) = T[x_2(t)] = x_2^2(t)$

$T [a_1 x_1(t) + a_2 x_2(t)] = [a_1 x_1(t) + a_2 x_2(t)]^2$

Which is not equal to $a_1 y_1(t) + a_2 y_2(t)$. Hence the system is said to be non linear.

Time Variant and Time Invariant Systems

A system is said to be time variant if its input and output characteristics vary with time. Otherwise, the system is considered as time invariant.

The condition for time invariant system is:

$$y (n , t) = y(n-t)$$

The condition for time variant system is:

$$y (n , t) \neq y(n-t)$$

Where $y (n , t) = T[x(n-t)]$ = input change

$y (n-t)$ = output change

Example:

$y(n) = x(-n)$

$y(n, t) = T[x(n-t)] = x(-n-t)$

$y(n-t) = x(-(n-t)) = x(-n + t)$

$\therefore$ $y(n, t) \neq y(n-t)$. Hence, the system is time variant.

Liner Time variant (LTV) and Liner Time Invariant (LTI) Systems

If a system is both liner and time variant, then it is called liner time variant (LTV) system.

If a system is both liner and time Invariant then that system is called liner time invariant (LTI) system.

Static and Dynamic Systems

Static system is memory-less whereas dynamic system is a memory system.

Example 1: $y(t) = 2\ x(t)$

For present value t=0, the system output is $y(0) = 2x(0)$. Here, the output is only dependent upon present input. Hence the system is memory less or static.

Example 2: $y(t) = 2\ x(t) + 3\ x(t-3)$

For present value t=0, the system output is $y(0) = 2x(0) + 3x(-3)$.

Here $x(-3)$ is past value for the present input for which the system requires memory to get this output. Hence, the system is a dynamic system.

Causal and Non-Causal Systems

A system is said to be causal if its output depends upon present and past inputs, and does not depend upon future input.

For non causal system, the output depends upon future inputs also.

Example 1: $y(n) = 2\ x(t) + 3\ x(t-3)$

For present value t=1, the system output is $y(1) = 2x(1) + 3x(-2)$.

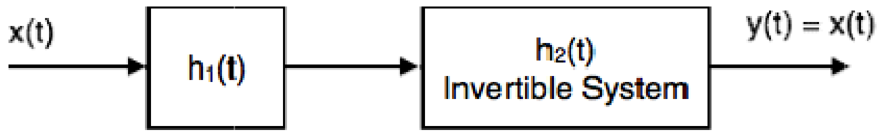
Here, the system output only depends upon present and past inputs. Hence, the system is causal.

Example 2: y(n) = 2 x(t) + 3 x(t-3) + 6x(t + 3)

For present value t=1, the system output is y(1) = 2x(1) + 3x(-2) + 6x(4) Here, the system output depends upon future input. Hence the system is non-causal system.

Invertible and Non-Invertible systems

A system is said to invertible if the input of the system appears at the output.



$$Y(S) = X(S)\ H1(S)\ H2(S)$$

$$= X(S)\ H1(S) \cdot 1(H1(S))$$

Since H2(S) = 1/(H1(S))

$$\therefore Y(S) = X(S)$$

$$\rightarrow y(t) = x(t)$$

Hence, the system is invertible.

If y(t) $\neq$ x(t), then the system is said to be non-invertible.

Stable and Unstable Systems

The system is said to be stable only when the output is bounded for bounded input. For a bounded input, if the output is unbounded in the system then it is said to be unstable.

Note: For a bounded signal, amplitude is finite.

Example 1: y (t) = x^2(t)

Let the input is u(t) (unit step bounded input) then the output y(t) = u2(t) = u(t) = bounded output.

Hence, the system is stable.

Example 2: y (t) = $\int x(t)dt$

Let the input is u (t) (unit step bounded input) then the output y(t) = $\int u(t)dt$ = ramp signal (unbounded because amplitude of ramp is not finite it goes to infinite when t $\longrightarrow$ infinite).

Hence, the system is unstable.

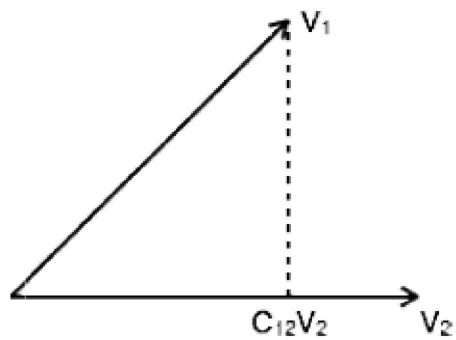
Analogy Between Vectors and Signals:

There is a perfect analogy between vectors and signals.

Vector

A vector contains magnitude and direction. The name of the vector is denoted by bold face type and their magnitude is denoted by light face type.

Example: V is a vector with magnitude V. Consider two vectors V_1 and V_2 as shown in the following diagram. Let the component of V_1 along with V_2 is given by $C_{12}V_2$. The component of a vector V_1 along with the vector V_2 can obtained by taking a perpendicular from the end of V_1 to the vector V_2 as shown in diagram:



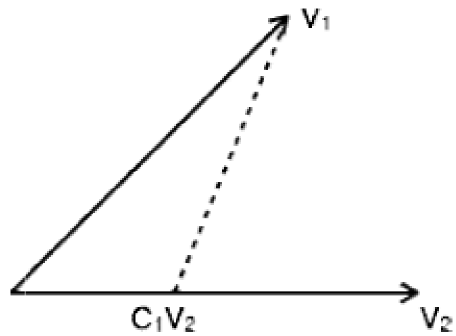
The vector V_1 can be expressed in terms of vector V_2

$$V_1 = C_{12}V_2 + V_e$$

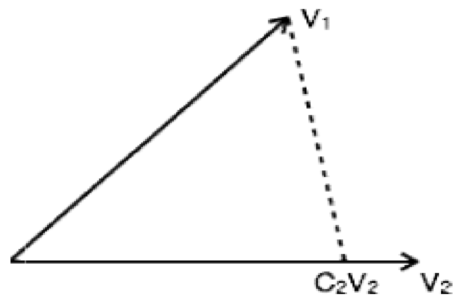
Where Ve is the error vector.

But this is not the only way of expressing vector V_1 in terms of V_2. The alternate possibilities are:

$V_1 = C_1V_2 + V_{e1}$



$V_2 = C_2V_2 + V_{e2}$



The error signal is minimum for large component value. If $C_{12}=0$, then two signals are said to be orthogonal.

Dot Product of Two Vectors

$V_1 . V_2 = V_1.V_2 \cos\theta$

θ = Angle between V1 and V2

$V_1. V_2 = V_2.V_1$

From the diagram, components of V_1 a long $V_2 = C_{12} V_2$

$$\frac{V_1 . V_2}{V_2 = C_1 2\, V_2}$$

$$\Rightarrow C_{12} = \frac{V_1 . V_2}{V_2}$$

Signal

The concept of orthogonality can be applied to signals. Let us consider two signals $f_1(t)$ and $f_2(t)$. Similar to vectors, you can approximate $f_1(t)$ in terms of $f_2(t)$ as

$f_1(t) = C_{12} f_2(t) + f_e(t)$ for $(t_1 < t < t_2)$

$\Rightarrow f_e(t) = f_1(t) - C_{12} f_2(t)$

One possible way of minimizing the error is integrating over the interval t_1 to t_2.

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [f_e(t)] dt$$

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [f_1(t) - C_{12} f_2(t)] dt$$

However, this step also does not reduce the error to appreciable extent. This can be corrected by taking the square of error function.

$$\varepsilon = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [f_e(t)]^2 dt$$

$$\Rightarrow \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [f_e(t) - C_{12} f_2]^2 dt$$

Where ε is the mean square value of error signal. The value of C_{12} which minimizes the error, you need to calculate $d\varepsilon/dC12=0$

$$\Rightarrow \frac{d}{dC_{12}} [\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [f_1(t) - C_{12} f_2(t)]^2 dt] = 0$$

$$\Rightarrow \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [\frac{d}{dC_{12}} f_1^2(t) - \frac{d}{dC_{12}} 2 f_1(t) C_{12} f_2(t) + \frac{d}{dC_{12}} f_2^2(t) C_{12}^2] dt = 0$$

Derivative of the terms which do not have C12 term are zero.

$$\Rightarrow \int_{t_1}^{t_2} -2 f_1(t) f_2(t) dt + 2C_{12} \int_{t_1}^{t_2} [f_2^2(t)] dt = 0$$

If $C_{12} = \frac{\int_{t_1}^{t_2} f_1(t) f_2(t) dt}{\int_{t_1}^{t_2} f_2^2(t) dt}$ component is zero, then two signals are said to be orthogonal.

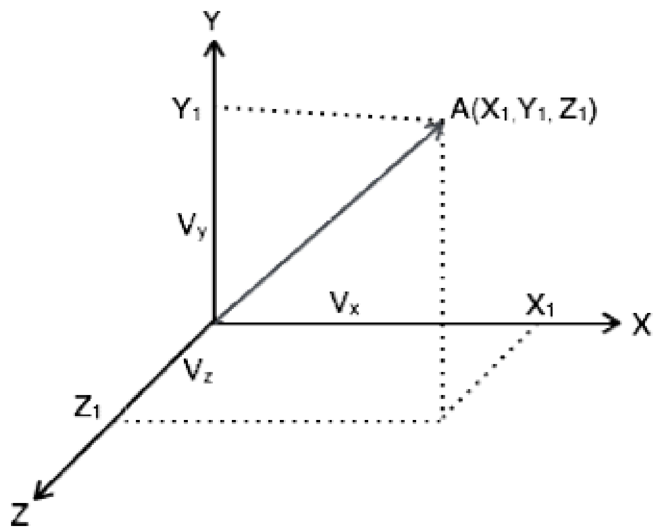
Put $C_{12} = 0$ to get condition for orthogonality.

$$0 = \frac{\int_{t_1}^{t_2} f_1(t)f_2(t)dt}{\int_{t_1}^{t_2} f_2^2(t)dt}$$

$$\int_{t_1}^{t_2} f_1(t)f_2(t)dt = 0$$

Orthogonal Vector Space

A complete set of orthogonal vectors is referred to as orthogonal vector space. Consider a three dimensional vector space as shown below:



Consider a vector A at a point (X_1, Y_1, Z_1). Consider three unit vectors (V_X, V_Y, V_Z) in the direction of X, Y, Z axis respectively. Since these unit vectors are mutually orthogonal, it satisfies that

$$V_X . V_X = V_Y . V_Y = V_Z . V_Z = 1$$
$$V_X . V_Y = V_Y . V_Z = V_Z . V_X = 0$$

We can write above conditions as

$$V_a \cdot V_b = \begin{cases} 1 & a = b \\ 0 & a \neq b \end{cases}$$

The vector A can be represented in terms of its components and unit vectors as

$$A = X_1 V_X + Y_1 V_Y + Z_1 V_Z \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Any vectors in this three dimensional space can be represented in terms of these three unit vectors only.

If you consider n dimensional space, then any vector A in that space can be represented as

$$A = X_1 V_X + Y_1 V_Y + Z_1 V_Z + \ldots + N_1 V_N \ldots\ldots (2)$$

As the magnitude of unit vectors is unity for any vector A

The component of A along x axis = $A.V_X$
The component of A along Y axis = $A.V_Y$
The component of A along Z axis = $A.V_Z$

Similarly, for n dimensional space, the component of A along some G axis

$$= A.V_G \ldots\ldots\ldots\ldots (3)$$

Substitute equation 2 in equation 3.

$$\Rightarrow CG = (X_1 V_X + Y_1 V_Y + Z_1 V_Z + \ldots + G_1 V_G \ldots + N_1 V_N) V_G$$

$$= X_1 V_X V_G + Y_1 V_Y V_G + Z_1 V_Z V_G + \ldots + G_1 V_G V_G \ldots + N_1 V_N V_G$$

$$= G_1 \quad \text{since } V_G V_G = 1$$

$$If V_G V_G \neq 1 \text{ i.e.} V_G V_G = k$$

$$AV_G = G_1 V_G V_G = G_1 K$$

$$G_1 = \frac{(AV_G)}{K}$$

Orthogonal Signal Space

Let us consider a set of n mutually orthogonal functions $x_1(t)$, $x_2(t)$... $x_n(t)$ over the interval t_1 to t_2. As these functions are orthogonal to each other, any two signals $x_j(t)$, $x_k(t)$ have to satisfy the orthogonality condition. i.e.

$$\int_{t_1}^{t_2} x_j(t)x_k(t)dt = 0 \ \text{ where } j \neq k$$

$$\text{Let} \int_{t_1}^{t_2} x_k^2(t)dt = k_k$$

Let a function f(t), it can be approximated with this orthogonal signal space by adding the components along mutually orthogonal signals i.e.

$$f(t) = C_1x_1(t) + C_2x_2(t) + \ldots + C_nx_n(t) + f_e(t)$$

$$= \Sigma_{r=1}^{n} C_r x_r(t)$$

$$f(t) = f(t) - \Sigma_{r=1}^{n} C_r x_r(t)$$

Mean sqaure error $\varepsilon = \frac{1}{t_2 - t_2} \int_{t_1}^{t_2} [f_e(t)]^2 dt$

$$= \frac{1}{t_2 - t_2} \int_{t_1}^{t_2} [f[t] - \sum_{r=1}^{n} C_r x_r(t)]^2 dt$$

The component which minimizes the mean square error can be found by

$$\frac{d\varepsilon}{dC_1} = \frac{d\varepsilon}{dC_2} = \ldots = \frac{d\varepsilon}{dC_k} = 0$$

Let us consider $\frac{d\varepsilon}{dC_k} = 0$

$$\frac{d}{dC_k}[\frac{1}{t_2-t_1}\int_{t_1}^{t_2}[f(t)-\Sigma_{r=1}^{n}C_r x_r(t)]^2 dt]=0$$

All terms that do not contain C_k is zero. i.e. in summation, r=k term remains and all other terms are zero.

$$\int_{t_1}^{t_2} -2f(t)x_k(t)dt + 2C_k\int_{t_1}^{t_2}[x_k^2(t)]dt = 0$$

$$\Rightarrow C_k = \frac{\int_{t_1}^{t_2} f(t)x_k(t)dt}{int_{t_1}^{t_2} x_k^2(t)dt}$$

$$\Rightarrow \int_{t_1}^{t_2} f(t)x_k(t)dt = C_k K_k$$

Mean Square Error:

The average of square of error function $f_e(t)$ is called as mean square error. It is denoted by ε (epsilon).

$$\varepsilon = \frac{1}{t_2-t_1}\int_{t_1}^{t_2}[f_e(t)]^2 dt$$

$$= \frac{1}{t_2-t_1}\int_{t_1}^{t_2}[f_e(t)-\Sigma_{r=1}^{n}C_r x_r(t)]^2 dt$$

$$= \frac{1}{t_2-t_1}[\int_{t_1}^{t_2}[f_e^2(t)]dt + \Sigma_{r=1}^{n}C_r^2\int_{t_1}^{t_2} x_r^2(t)dt - 2\Sigma_{r=1}^{n}C_r\int_{t_1}^{t_2} x_r(t)f(t)dt$$

You know that $C_r^2\int_{t_1}^{t_2} x_r^2(t)dt = C_r\int_{t_1}^{t_2} x_r(t)f(d)dt = C_r^2 K_r$

$$\varepsilon = \frac{1}{t_2-t_1}[\int_{t_1}^{t_2}[f^2(t)]dt + \Sigma_{r=1}^{n}C_r^2 K_r - 2\Sigma_{r=1}^{n}C_r^2 K_r]$$

$$= \frac{1}{t_2-t_1}[\int_{t_1}^{t_2}[f^2(t)]dt - \Sigma_{r=1}^{n}C_r^2 K_r]$$

$$\therefore \varepsilon = \frac{1}{t_2-t_1}[\int_{t_1}^{t_2}[f^2(t)]dt + (C_1^2K_1 + C_2^2K_2 + \ldots + C_n^2K_n)]$$

The above equation is used to evaluate the mean square error.

Closed and Complete Set of Orthogonal Functions:

Let us consider a set of n mutually orthogonal functions $x_1(t)$, $x_2(t)...x_n(t)$ over the interval t_1 to t_2. This is called as closed and complete set when there exist no function f(t) satisfying the condition

$$\int_{t_1}^{t_2} f(t)x_k(t)dt = 0$$

If this function is satisfying the equation

$$\int_{t_1}^{t_2} f(t)x_k(t)dt = 0$$

For k=1,2,.. then f(t) is said to be orthogonal to each and every function of orthogonal set. This set is incomplete without f(t). It becomes closed and complete set when f(t) is included.

f(t) can be approximated with this orthogonal set by adding the components along mutually orthogonal signals i.e.

$$f(t) = C_1x_1(t) + C_2x_2(t) + \ldots + C_nx_n(t) + f_e(t)$$

If the infinite series $C_1x_1(t) + C_2x_2(t) + \ldots + C_nx_n(t)$ converges to ft then mean square error is zero.

Orthogonality in Complex Functions:

If $f_1(t)$ and $f_2(t)$ are two complex functions, then $f_1(t)$ can be expressed in terms of $f_2(t)$ as

$f_1(t)=C_{12}f_2(t)$.. with negligible error

$$\text{Where } C_{12} = \frac{\int_{t_1}^{t_2} f_1(t)f_2^*(t)dt}{\int_{t_1}^{t_2} |f_2(t)|^2 dt}$$

Where $f_2^*(t)$ is the complex conjugate of $f_2(t)$

If $f_1(t)$ and $f_2(t)$ are orthogonal then $C_{12} = 0$

$$\frac{\int_{t_1}^{t_2} f_1(t) f_2^*(t) dt}{\int_{t_1}^{t_2} |f_2(t)|^2 dt} = 0$$

$$\Rightarrow \int_{t_1}^{t_2} f_1(t) f_2^*(dt) = 0$$

The above equation represents orthogonality condition in complex functions.

Fourier series:

To represent any periodic signal x(t), Fourier developed an expression called Fourier series. This is in terms of an infinite sum of sines and cosines or exponentials. Fourier series uses orthoganality condition.

Jean Baptiste Joseph Fourier,a French mathematician and a physicist; was born in Auxerre, France. He initialized Fourier series, Fourier transforms and their applications to problems of heat transfer and vibrations. The Fourier series, Fourier transforms and Fourier's Law are named in his honour.

Fourier Series Representation of Continuous Time Periodic Signals

A signal is said to be periodic if it satisfies the condition x (t) = x (t + T) or x (n) = x (n + N).

Where T = fundamental time period,

ω_0= fundamental frequency = 2π/T

There are two basic periodic signals:
$x(t)=\cos\omega_0 t$(sinusoidal) &
$x(t)=e^{j\omega_0 t}$(complex exponential)
These two signals are periodic with period $T=2\pi/\omega_0$

. A set of harmonically related complex exponentials can be represented as {$\phi_k(t)$}

$$\phi_k(t) = \{e^{jk\omega_0 t}\} = \{e^{jk(\frac{2\pi}{T})t}\} \text{where}\, k = 0 \pm 1, \pm 2..n \;\; .....(1)$$

All these signals are periodic with period T

According to orthogonal signal space approximation of a function x (t) with n, mutually orthogonal functions is given by

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \ldots\ldots (2)$$

$$= \sum_{k=-\infty}^{\infty} a_k k e^{jk\omega_0 t}$$

Where a_k = Fourier coefficient = coefficient of approximation.

This signal x(t) is also periodic with period T.

Equation 2 represents Fourier series representation of periodic signal x(t).

The term k = 0 is constant.

The term $k=\pm 1$ having fundamental frequency ω_0 , is called as 1^{st} harmonics.

The term $k=\pm 2$ having fundamental frequency $2\omega_0$, is called as 2^{nd} harmonics, and so on...

The term $k=\pm n$ having fundamental frequency $n\omega_0$, is called as n^{th} harmonics.

Deriving Fourier Coefficient

We know that

$$x(t) = \Sigma_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \ldots\ldots (1)$$

Multiply $e^{-jn\omega_0 t}$ on both sides. Then

$$x(t)e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} . e^{-jn\omega_0 t}$$

Consider integral on both sides.

SIGNALS and SYSTEMS

UNIT - I : Introduction : Definition of Signals and Systems, classification of Signals, Classification of Systems, Operations on signals : time - shifting, time - scaling, amplitude - shifting, amplitude - scaling. Problems on classification and characteristics of signals and systems. Complex exponential and sinusoidal signals, Singularity functions and related functions : impulse function, step function, signum function and ramp function. Analogy between vectors and signals, orthogonal signal space, Signal approximation using orthogonal functions, Mean Square error, closed or complete set of orthogonal functions, Orthogonality in complex functions, Related problems.

UNIT-II : Fourier Series and Fourier Transform :

Fourier series representation of continuous time periodic signals, properties of Fourier series, Dirichlet's conditions, Trigonometric Fourier series and Exponential Fourier series, Relation between Trigonometric and Exponential Fourier series, Complex Fourier spectrum. Deriving Fourier transform from Fourier series, Fourier transform of arbitrary signal, Fourier transform of standard signals, Fourier transform of periodic signals, properties of Fourier transforms, Fourier transforms involving impulse function and Signum function. Introduction to Hilbert Transform. Related Problems.

UNIT-III : Analysis of linear systems : Introduction, Linear System, impulse response, Response of a linear system, Linear time invariant (LTI) system, Linear time variant (LTV) system, Concept of convolution in time domain and frequency domain, Graphical representation of convolution, Transfer function of a LTI system, Related problems, Filter characteristics of linear systems. Distortion less transmission through a system, Signal bandwidth, system bandwidth, Ideal LPF, HPF and BPF characteristics, Causality and poly - Wiener criterion for physical realization, relationship between bandwidth and rise time.

UNIT - IV :-

Correlation :- Auto-correlation and cross-correlation of functions, properties of correlation function, Energy density spectrum, Parseval's theorem, Power density spectrum, Relation between Convolution and correlation, Detection of periodic signals in the presence of noise by correlation, Extraction of signal from noise by filtering.

Sampling Theorem :- Graphical and analytical proof for Band Limited Signals, impulse sampling, Natural and Flat top sampling, Reconstruction of signal from its samples, effect of under sampling - Aliasing, Introduction to Band Pass sampling, Related problems.

UNIT - V :-

Laplace Transforms :- Introduction, Concept of region of convergence (ROC) for laplace transforms, constraints on ROC for various classes of signals, Properties of L.T's Inverse laplace transform, Relation between L.T's, and F.T. of a signal. Laplace transform of certain signals using waveform synthesis.

Z - Transforms : Concept of Z - Transform of a discrete sequence. Region of convergence in Z - Transform, constraints on ROC for various classes of signals, Inverse Z - transform, properties of Z - transform, Distinction between Laplace, Fourier and Z - transforms.

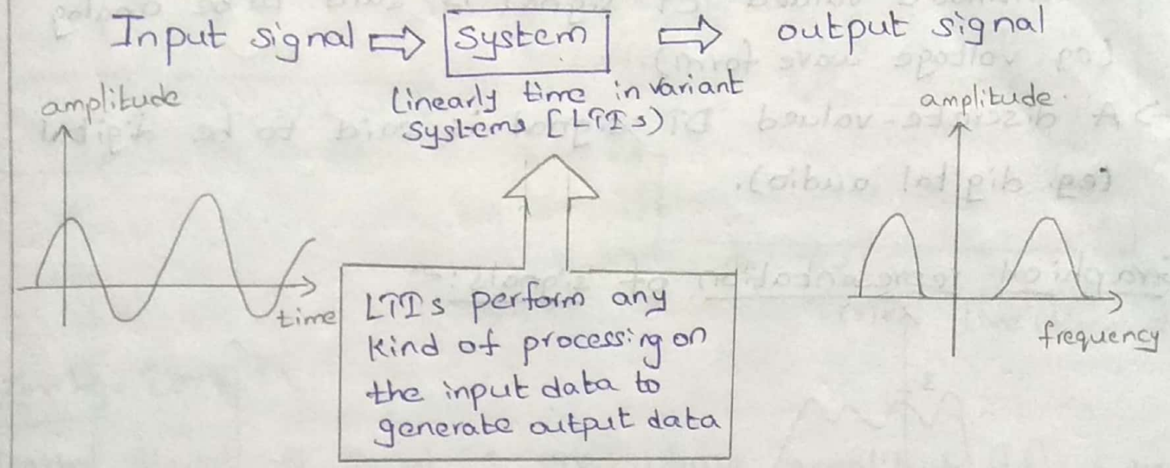
UNIT - I

INTRODUCTION

Signal:- A signal is defined as a time varying physical phenomenon which is intended to convey information (or) Signal is a function of time (or) signal is a function of one or more independent variables which contain some information.

Eg:- Voice signal, Video signal, signals on telephon wires EEG, ECG etc.

Signals may be of continuous time or discrete time signal.



Signals:-

* A signal is a function of one or more variables that conveys information about some (usually physical) phenomenon.
* For a function f, in the expansion $f(t_1, t_2, t_3 \dots t_n)$ each of the $\{t_k\}$ is called an independent variable, while the function value itself is referred to as a dependent variables.
* Some examples include:
  - → A voltage or current in an electronic circuit.
  - → The position, velocity, or accelaration, of a object
  - → A force or torque in a mechanical system
  - → A flow rate of a liquid or gas in a chemical process
  - → A digital image, digital video, or digital audio.
  - → A stock market index.

Classification of signals :-

→ Number of independent variables (i.e., dimensionality)

* A signal with one independent variable is said to be one dimensional (e.g. audio)
* A signal with more than one independent variable is said to be multi dimensional (e.g image)

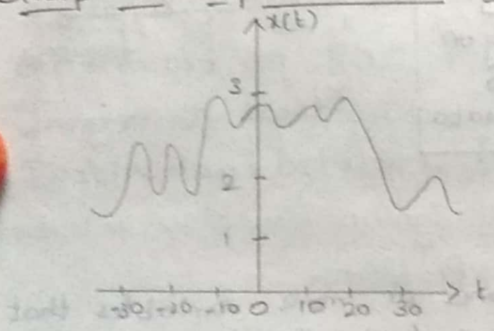
→ Continuous or discrete independent variables.

* A signal with continuous independent variables is said to be continuous time(CT) (eg. voltage waveform)
* A signal with discrete independent variables is said to be discrete time (DT) (eg. stock market index)

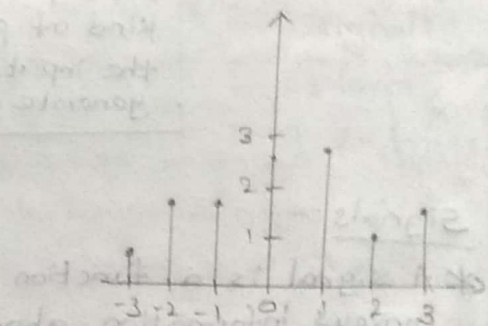
→ A continuous-valued CT signal is said to be analog (eg. voltage wave form)

→ A discrete-valued DT signal is said to be digital (eg. digital audio).

Graphical representation of signals :-



Continuous-Time (CT) signal



Discrete-Time (DT) signal

Classification of Signals :-

→ Continuous time - Discrete time
→ Analog - Digital (numerical)
→ Periodic - aperiodic
→ Energy - power
→ Deterministic - Random (Probabilistic)

Note :- Such classes are not disjoint, so there are digital signals that are periodic of power-type and others that are aperiodic of power type etc.

→ Any Combination of signals features from the different classes is possible.

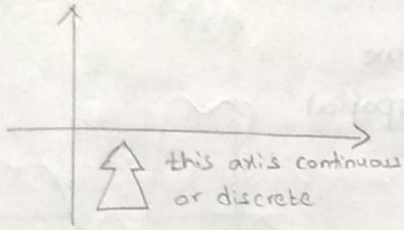
Continuous time - Discrete time :-

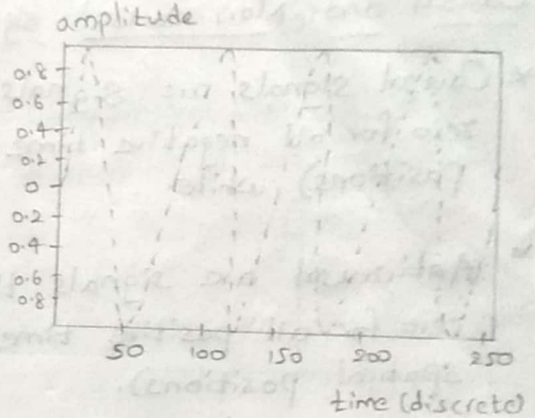
Discrete time signal:- A signal that is specified only for discrete values of the independent variable.

-> It is usually generated by sampling so it will only have values at equally spaced intervals along the time axis.

-> The domain of the function representing the signal has cardinality of integer numbers.

* Signal <-> f = f [n], also called "sequence"
* Independent variable <-> n
* For discrete-time functions $t \in Z$





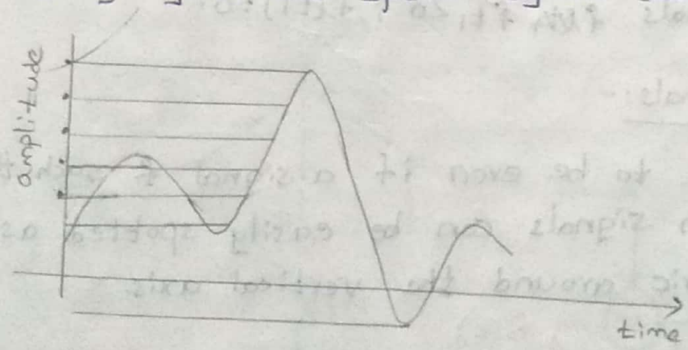
-> Analog - Digital

Digital Signal:- A signal is one whose amplitude can take on only a infinite number of values (thus it is quantized).

-> The amplitude of the function f( ) can take only a finite number of values.

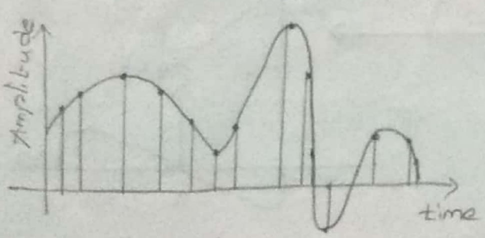
-> A digital signal whose amplitude can take only M different values is said to be M-ary

- Binary signals are special signals case for M=2.

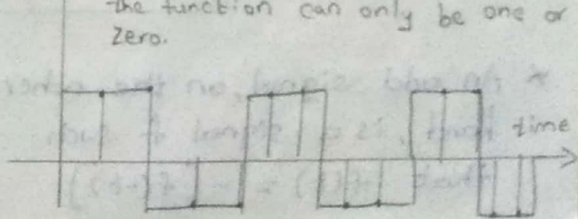


Example:-

* Discrete time Analog



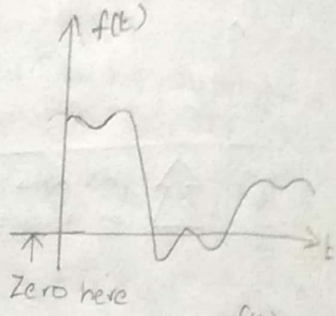
* Discrete time digital
  - Binary sequence where the values of the function can only be one or Zero.



Summary :-

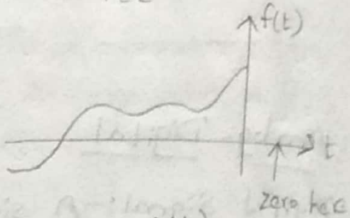
Signal amplitude / Time or space	Real	Integer
Real	Analog Continuous-time	Digital Continuous-time
Integer	Analog Discrete-time	Digital Discrete time

Causal and Non-causal signals :-

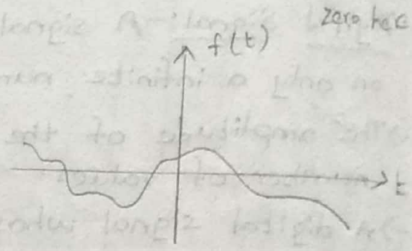


* Causal signals are signals that are zero for all negative time (or spatial positions), while.

* Anticausal are signals that are zero for all positive time (or spatical positions).



* Noncausal signals are signals that have nonzero values in both positive and negative time.



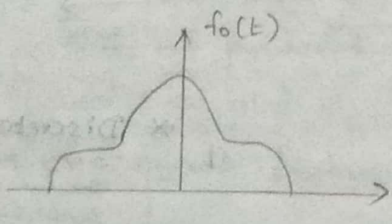
i.e., Causal signals $f(t) = 0,\ t < 0$

→ Anti causal signals $f(t) = 0,\ t \geq 0$

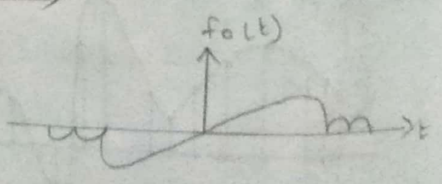
→ Non-Causal signals $f\ t_1 < 0\ ;\ f(t_1) = 0$.

Even and Odd Signals :-

* A signal is said to be even if a signal f such that $f(t) = f(-t)$. Even signals can be easily spotted as they are symmetric around the vertical axis.



* An odd signal, on the other hand, is a signal f such that $f(t) = -[f(-t)]$



Decomposition in even and odd Components:-

* Any signal can be written as a combination of an even and an odd signals

- Even and odd signals

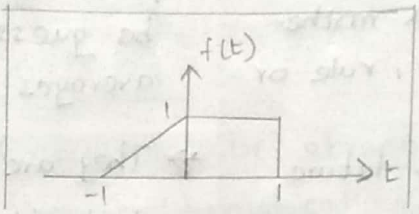
$$f(t) = \frac{1}{2}\left(f(t) + f(-t)\right) + \frac{1}{2}\left(f(t) - f(-t)\right)$$

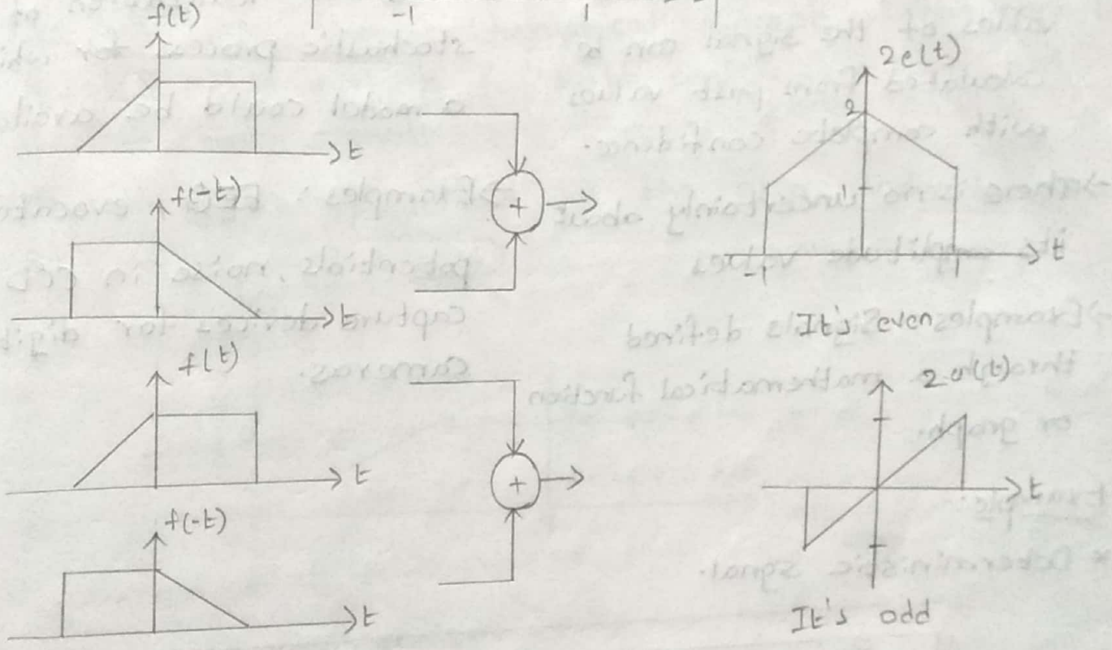
$$f_e(t) = \frac{1}{2}\left(f(t) + f(-t)\right) \text{ even component}$$

$$f_o(t) = \frac{1}{2}\left(f(t) - f(-t)\right) \text{ odd component}$$

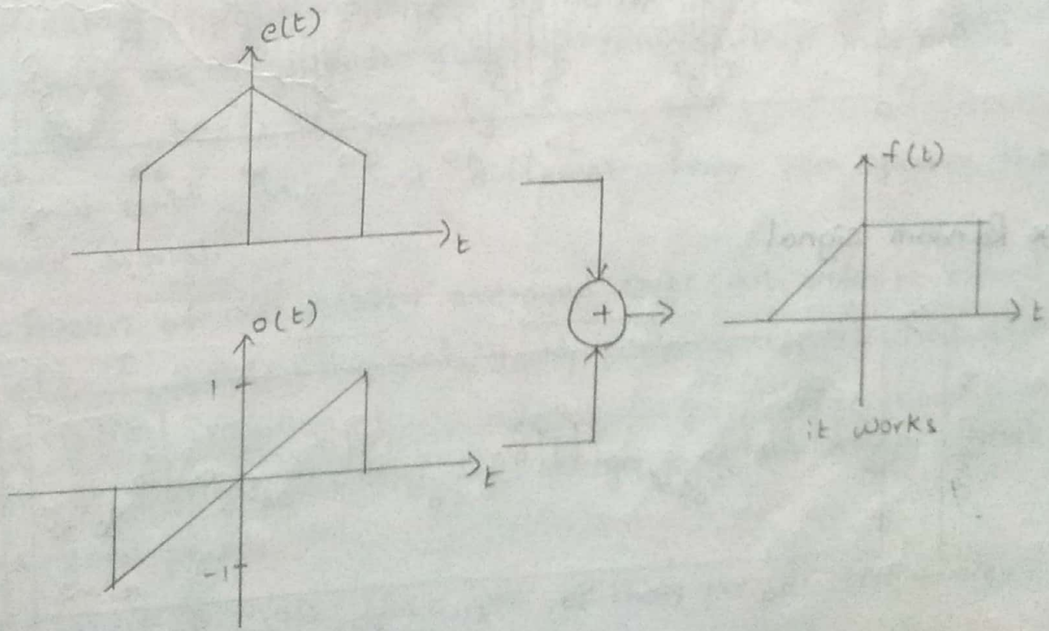
$$\therefore f(t) = f_e(t) + f_o(t).$$

Example:-





Proof:-



Deterministic - Probabilistic :-

* Deterministic signal : A signal whose physical description is known completely.

* A deterministic signal is a signal in which each value of the signal is fixed and can be determined by a mathematical expression, rule or table.

* Because of this the future values of the signal can be calculated from past values with complete confidence.

-> There is no uncertainly about its amplitude values

-> Examples : Signals defined through a mathematical function or graph.

* Probabilistic (OR) Random Signal :- The amplitude values cannot be predicted precisely but are known only in terms of probabilistic descriptions.

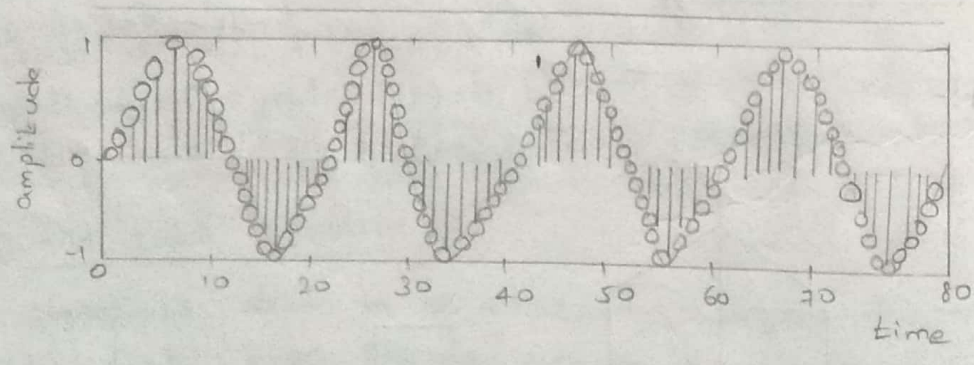
* The future values of a random signal cannot be accurately predicted and can usually only be guessed based on the averages of sets of signals.

* They are realization of a stochastic process for which a model could be available.

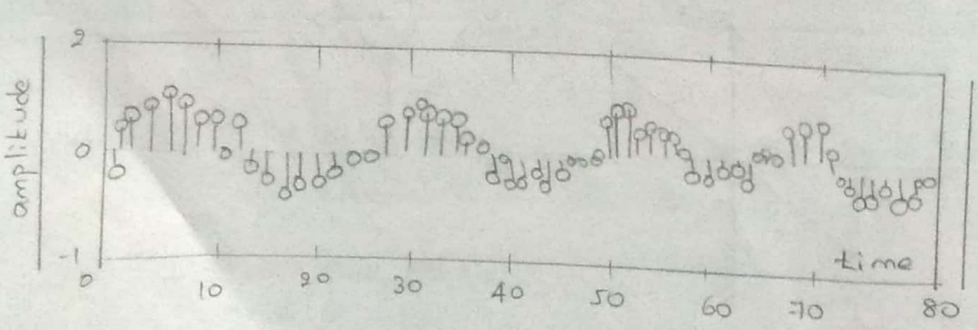
-> Examples : EEG, evocated potentials, noise in CCD capture devices for digital cameras.

Example :-

* Deterministic signal.



* Random signal



Finite and infinite length signals

* A finite length signal is non-zero over a finite set of values of the independent variable.

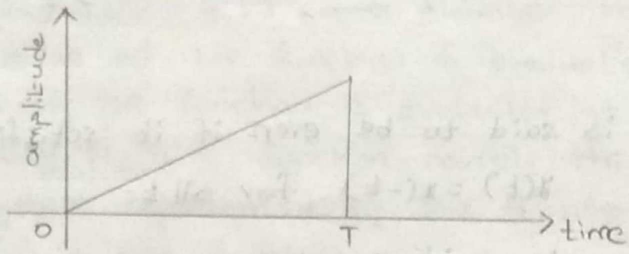
$$f = f(t),\ \forall t : t_1 \le t \le t_2$$

$$t_1 > -\infty,\ t_2 < +\infty$$

* An infinite length signal is non zero over an infinite set of values of the independent variable
  - For instance, a sinusoid $f(t) = \sin(\omega t)$ is an infinite length signal.

Size of a Signal: Norms

* "Size" indicates largeness or strength.
* We will use the mathematical concept of the norm to qualify this notion for both continuous-time and discrete-time signals.
* The energy is represented by the area under the curve (of the squared signal)



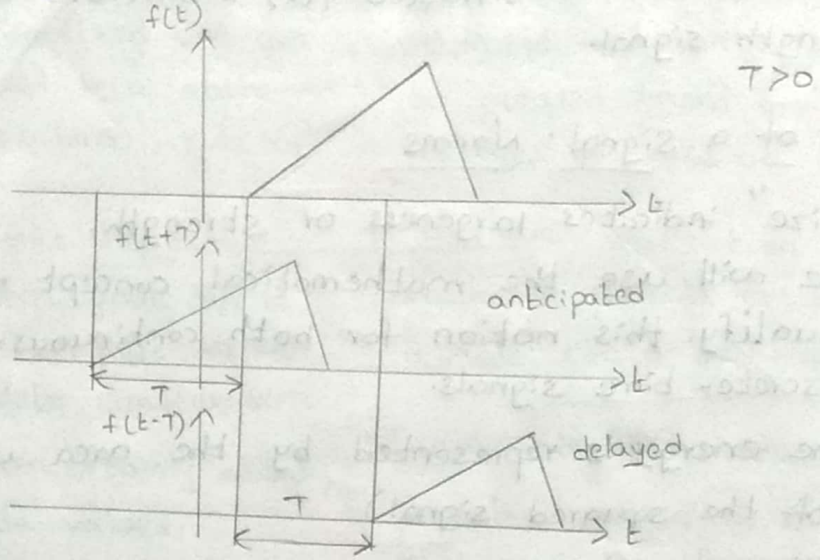
Energy and Power signals :-

* A signal with finite energy is an energy signal
  - Necessary condition for a signal to be of energy type is that the amplitude goes to zero as the independent variable tends to infinity.
* A signal with finite and different from zero power is a power signal.
  - The mean of an entity averaged over an infinite interval exists if either the entity is periodic or it has some statistical regularity.
  - A power signal has infinite energy and an energy signal has zero power.
  - There exist signals that are neither power nor energy, such as the ramp.

* All practical signals have finite energy and thus are energy signals.
- It is impossible to generate a real power signal because this would have infinite duration and infinite energy, which is not doable.

Useful signal operations: shifting, scaling, inversion

* Shifting: Consider a signal $f(t)$ and the same signal delayed/anticipated by T seconds.



Even Signals:-

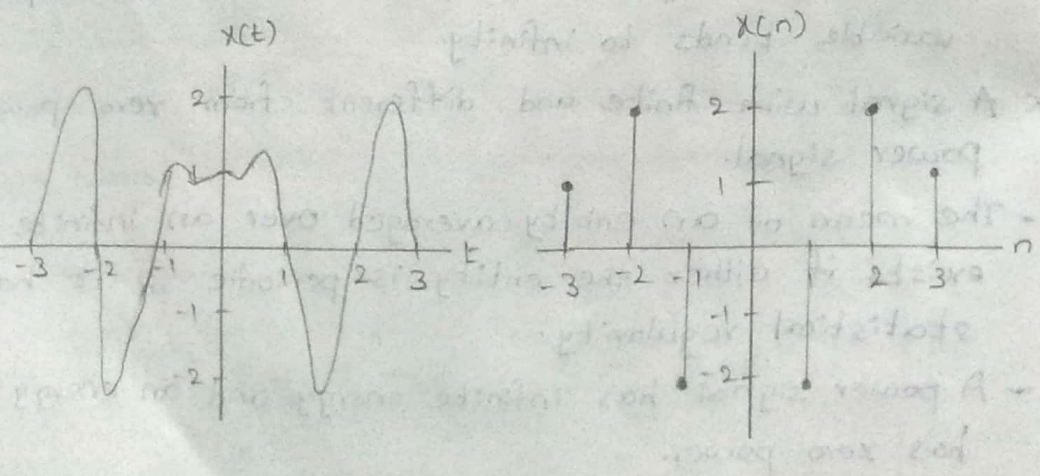
* A function x is said to be even if it satisfies

$$x(t) = x(-t) \quad \text{for all } t.$$

* A sequence x is said to be even if it satisfies

$$x(n) = x(-n) \quad \text{for all } n.$$

* Geometrically, the graph of an even signal is symmetric about the origin.
* Some examples of even signals are shown below.



Important points to Remember

* strictly an expression like "f(t)" means the value of the function f evaluated at point t.
* Unfortunately, engineers often ~~uge~~ use an expression like "f(t)" to refer to the function f (rather than the value of f evaluated at point t), and this sloppy notation can lead to problems (eg ambiguity) in some situations.
* In contexts where sloppy notation may lead to problems, one should be careful to clearly distinguish between a function and its value.
* Examples (meaning of notation)

-> Let f and g denote real-valued functions of a real variable.

-> Let t denote an arbitrary real number

-> Let H denote a system operator (which maps a function to a function)

-> The quantity f+g is a function namely the function formed by adding the functions f and g.

-> The quantity $f(t)+g(t)$ is a number, namely the sum of the value of the function f evaluated at t; and the value of the function g evaluated at t.

-> The quantity Hx is a function, namely the output produced by the system represented by the H when the input to the system is the function x.

-> The quantity $Hx(t)$ is a number, namely, the value of the function Hx evaluated at t.

Combined Time scaling and Time shifting :-

-> Consider a transformation that maps the input signal x to the output signal y as given by

$$y(t) = x(at-b)$$

Where a and b are real numbers and $a \neq 0$

-> The above transformation can be shown to the combination of a time scaling operation and time shifting operation.

-> Since time scaling and time shifting do not continue, we must be particularly careful about the order in which these transformation are applied.

-> The above transformation has two distinct but equivalent interpretation.

* first, time shifting x by b, and then time scaling the

result by a.

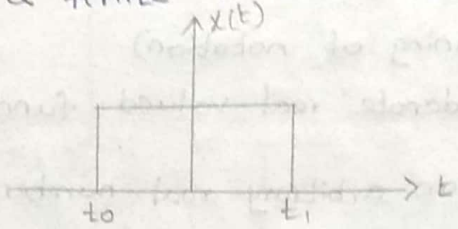
* first, time scaling x by a, and then time shifting result by b/a.

→ Note that time shift is not by the same in both cases.

Finite Duration and Two sided Signals :-

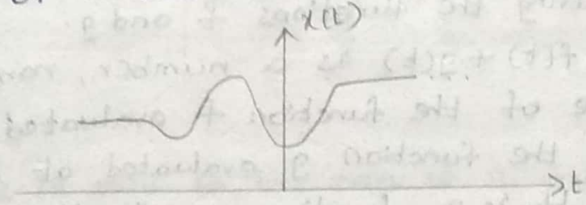
* A signal that is both left sided and right sided is said to be finite duration (or time limited).

* An example of a finite duration signal is shown below



* A signal that is neither left sided nor right sided is said to be two sided.

* An example of a two sided signal shown below.



Bounded Signals :-

* A signal x is said to be bounded if there exists some (finite) positive real constant A such that

$$|x(t)| \leq A \text{ for all } t.$$

(i.e., x(t) is finite for all t).

* Examples of bounded signals include the sine and cosine functions.

* Examples of unbounded signals include the tan function and any nonconstant polynomial function.

Signal Energy and Power :-

* The energy E contained in the signal x is given by

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

* A signal with finite energy is said to be an energy signal.

* The average power P contained in the signal x is given by

$$P = \lim_{T \to \infty} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

* A signal with (non zero) finite average power is said to be a power signal.

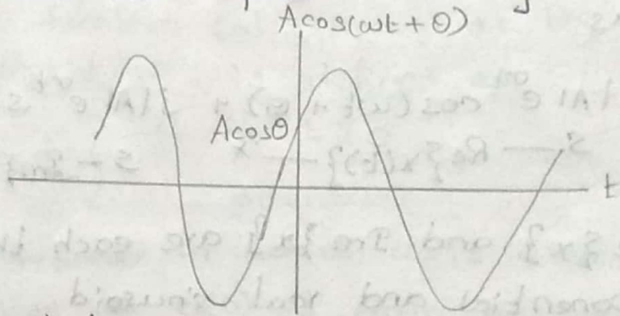
Real Sinusoids :-

* A (CT) real sinusoid is a function of the form

$$x(t) = A\cos(\omega t + \theta)$$

Where A, ω, θ are real constants.

* Such a function is periodic with fundamental period $T = \frac{2\pi}{|\omega|}$ and fundamental frequency $|\omega|$
* A real sinusoid has a plot resembling that shown below



Complex Exponentials :-

* A (CT) complex exponential is a function of the form

$$x(t) = Ae^{\lambda t},$$

Where A and λ are complex constants.

* A complex exponential can exhibit one of a number of distinct modes of behaviour, depending on the values of its parameters A and λ.
* For example, as special cases, complex exponential include a real exponentials and complex sinusoids.

Real Exponentials :-

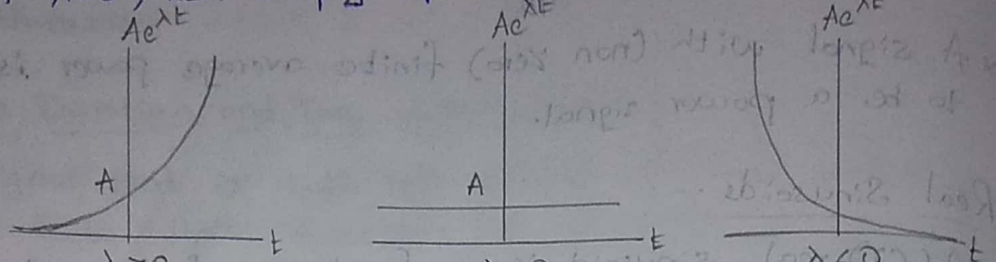
* A real exponential is a special case of a complex exponential $x(t) = Ae^{\lambda t}$, where A and λ are restricted to be real numbers.
* A real exponential can exhibit one of three distinct modes of behaviour depending on the value of λ, as illustrated below.

* If $\lambda > 0$, $x(t)$ increases exponentially as t increases

(i.e., a growing exponential), If

* If $\lambda < 0$, $x(t)$ decreases exponentially as t increases (i.e., a decay exponential).

* If $\lambda = 0$, $x(t)$ simply equals the constant A.



$\lambda > 0$ $\lambda = 0$ $\lambda < 0$

General Complex Exponentials

* In the most general case of a complex exponential $x(t) = Ae^{\lambda t}$, A and λ are both complex.

* Letting $A = |A| e^{j\theta}$ and $\lambda = \sigma + j\omega$ (where θ, σ and ω are real), and using Euler's relation, we can rewrite $x(t)$ as

$$x(t) = \underbrace{|A| e^{\sigma t} \cos(\omega t + \theta)}_{Re\{x(t)\}} + j \underbrace{|A| e^{\sigma t} \sin(\omega t + \theta)}_{Im\{x(t)\}}$$

* Thus, $Re\{x\}$ and $Im\{x\}$ are each the product of a real exponential and real sinusoid

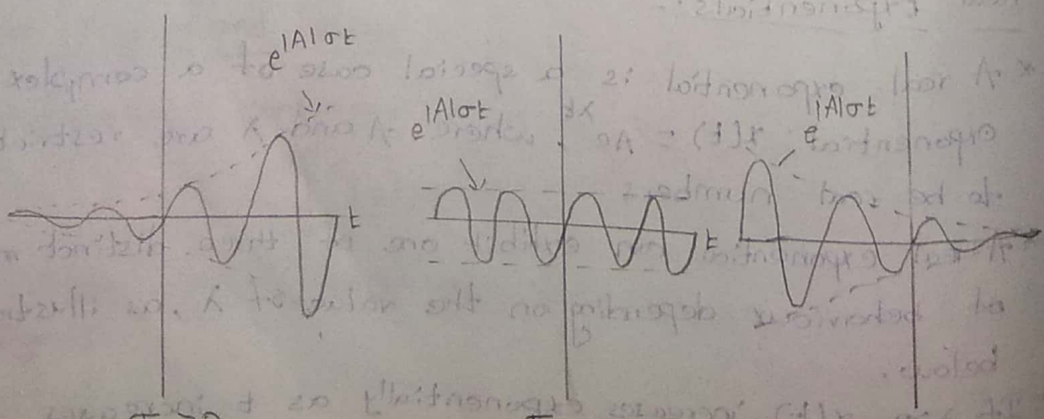
* One of three distinct modes of behaviour is exhibited by $x(t)$, depending on the value of σ.

* If $\sigma = 0$, $Re\{x\}$ and $Im\{x\}$ are real sinusoids.

* If $\sigma > 0$, $Re\{x\}$ and $Im\{x\}$ are each the product of a real sinusoid and a growing real exponential.

* If $\sigma < 0$, $Re\{x\}$ and $Im\{x\}$ are each the product of a real sinusoid and a decaying real exponential.

* The three modes of behaviour for $Re\{x\}$ and $Im\{x\}$ are illustrated below.



$\sigma > 0$ $\sigma = 0$ $\sigma < 0$

Relationship Between Complex Exponentials and Real Sinusoids:

* From Euler's relation, a complex sinusoid can be expressed as the sum of two real sinusoids as

$$Ae^{j\omega t} = A\cos\omega t + jA\sin\omega t.$$

* Moreover, a real sinusoid can be expressed as the sum of two complex sinusoids using the identities.

$$A\cos(\omega t+\theta) = \frac{A}{2}\left[e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}\right] \text{ and}$$

$$A\sin(\omega t+\theta) = \frac{A}{2j}\left[e^{j(\omega t+\theta)} - e^{-j(\omega t+\theta)}\right]$$

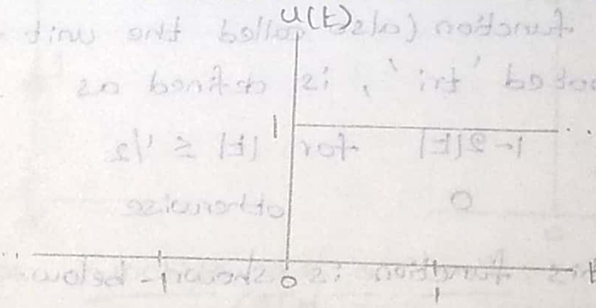
* Note that, above, we are simply restating results from the (appendix) material on complex analysis.

Unit Step Function:-

* The unit-step function (also known as the Heaviside function), denoted 'u', is defined as

$$u(t) = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

* Due to the manner in which u is used in practice, the actual value of u(0) is unimportant. Sometimes values of 0 and $\frac{1}{2}$ are also used for u(0).
* A plot of this function is shown below.

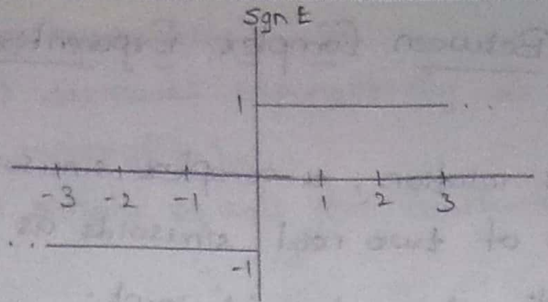


Signum function:-

* The signum function, denoted 'sgn', is defined as

$$\text{sgn}(t) = \begin{cases} 1 & \text{if } t > 0 \\ 0 & \text{if } t = 0 \\ -1 & \text{if } t < 0 \end{cases}$$

* From its definition, one can see that the signum function simply computes the sign of a number.
* A plot of this function is shown below.



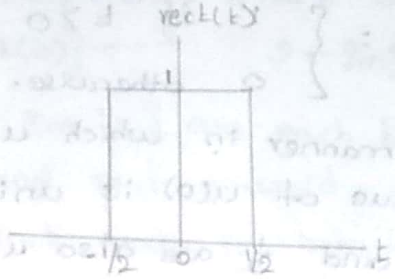
Rectangular Function :-

* The rectangular function (also called the unit-rectangular pulse function), denoted 'rect', is given by.

$$\text{rect}(t) = \begin{cases} 1 & \text{if } -\frac{1}{2} \le t < \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}$$

* Due to the manner in which the rect function is used in practice, the actual value of rect(t) at $t = \pm\frac{1}{2}$ is unimportant. Sometimes different values are used from those specified above.

* A plot of this function is shown below.

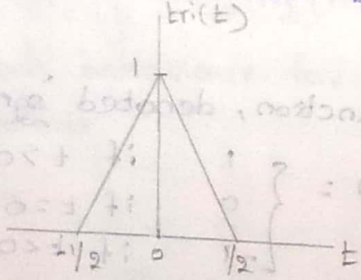


Triangular Function :-

* The triangular function (also called the unit-triangular pulse function) denoted 'tri', is defined as

$$\text{tri}(t) = \begin{cases} 1-2|t| & \text{for } |t| \le 1/2 \\ 0 & \text{otherwise} \end{cases}$$

* A plot of this function is shown below.



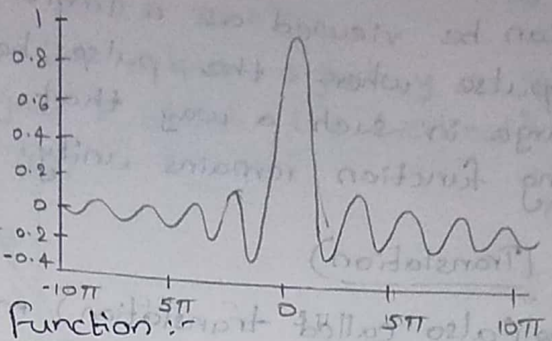
Cardinal Sine function :-

* The cardinal sine function, denoted 'sinc', is given by

$$\text{sinc}(t) = \frac{\sin t}{t}$$

* By l'Hopital's rule, sinc 0 = 1.

* A plot of this function for part of the real line is shown below.
[Note that the oscillations in sinc(t) do not die out for finite t.].



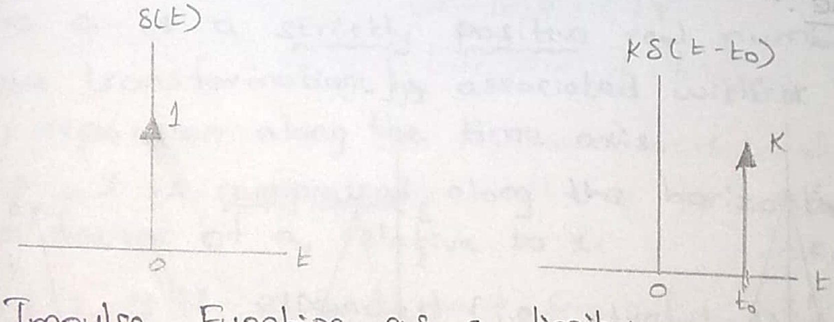
Unit-Impulse Function :-

* The unit-impulse function (also known as the Dirac delta function or delta function), denoted δ, is defined by the following two properties :

$\delta(t) = 0$ for $t \neq 0$ and $\delta(t) = \begin{cases} 1, & t = 0 \\ 0 & \text{otherwise} \end{cases}$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

* Technically, δ is not a function in the ordinary sense. Rather it was what is known as generalized function. Consequently, the δ function sometimes behaves in unusual ways.
* Graphically, the delta function is represented as shown below.

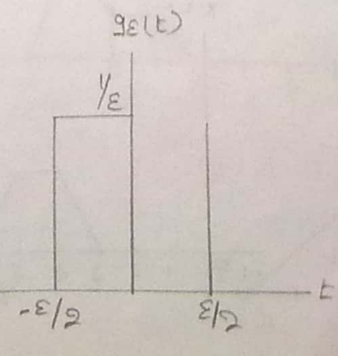


Unit-Impulse Function as a limit :-

* Define

$$g_\varepsilon(t) = \begin{cases} 1/\varepsilon & \text{for } |t| < \varepsilon/2 \\ 0 & \text{otherwise.} \end{cases}$$

* The function g_ε has a plot of the form shown below.



* Clearly, for any choice of ε, $\int g_\varepsilon(t)\,dt = 1$.
* The function δ can be obtained as the following limit.

$$\delta(t) = \lim_{\varepsilon \to 0} g_\varepsilon(t).$$

* That is, δ can be viewed as a limiting case of a rectangular pulse where the pulse height becomes infinitely large in such a way that the integral of the resulting function remains unity.

Time Shifting (Translation)

* Time shifting (also called translation) maps the input signal x to the output signal y as given by

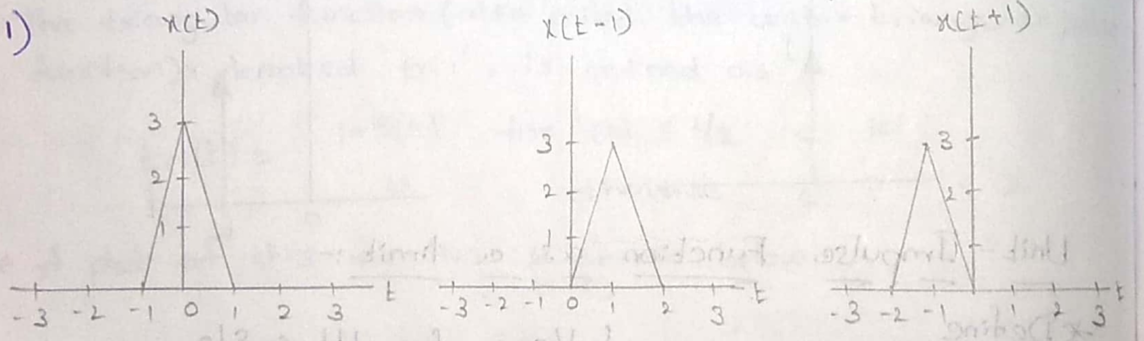
$$y(t) = x(t-b),$$

where b is a real number.

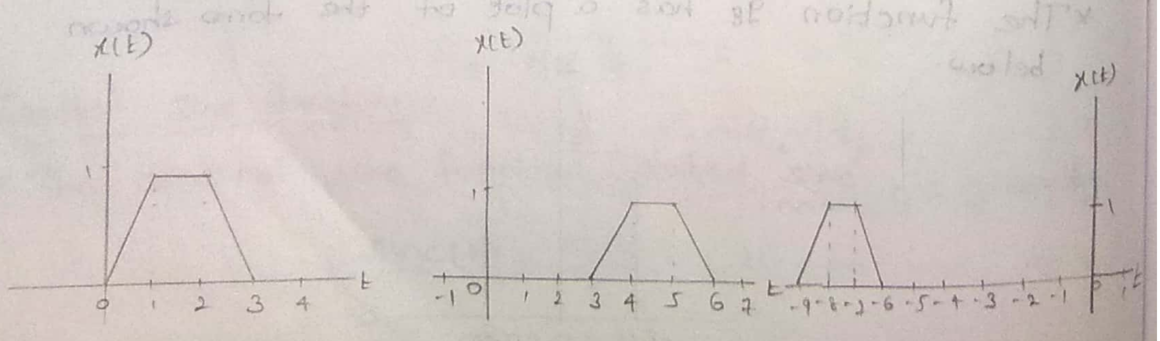
* Such a transformation shifts the signal (to the left or right) along the time axis.
* If $b>0$, y is shifted to the right by $|b|$, relative to x (i.e., delayed in time).
* If $b<0$, y is shifted to the left by $|b|$, relative to x (i.e., advanced in time).

Example :-

1)



2)



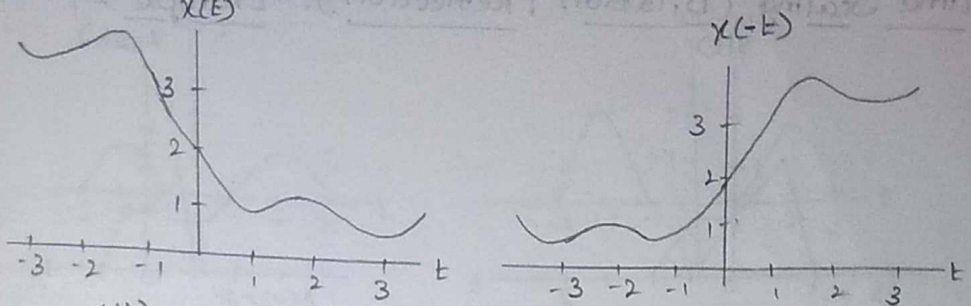
Time Reversal (Reflection)

* Time reversal (also known as reflection) maps the input signal x to the output signal y as given by

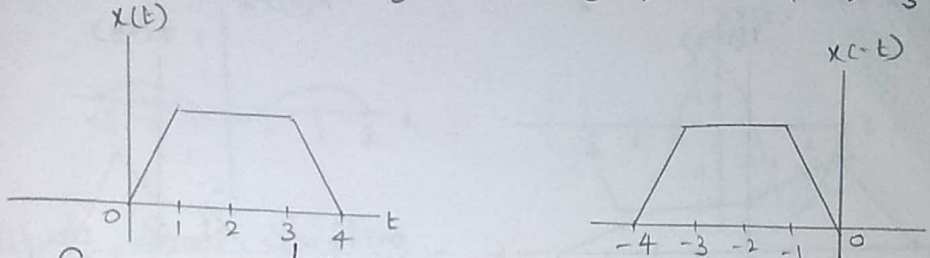
$$y(t) = x(-t).$$

* Geometrically, the output signal y is a reflection of the input signal x about the (vertical) line $t=0$.

1)



2)



Time Compression / Expansion (Dilation)

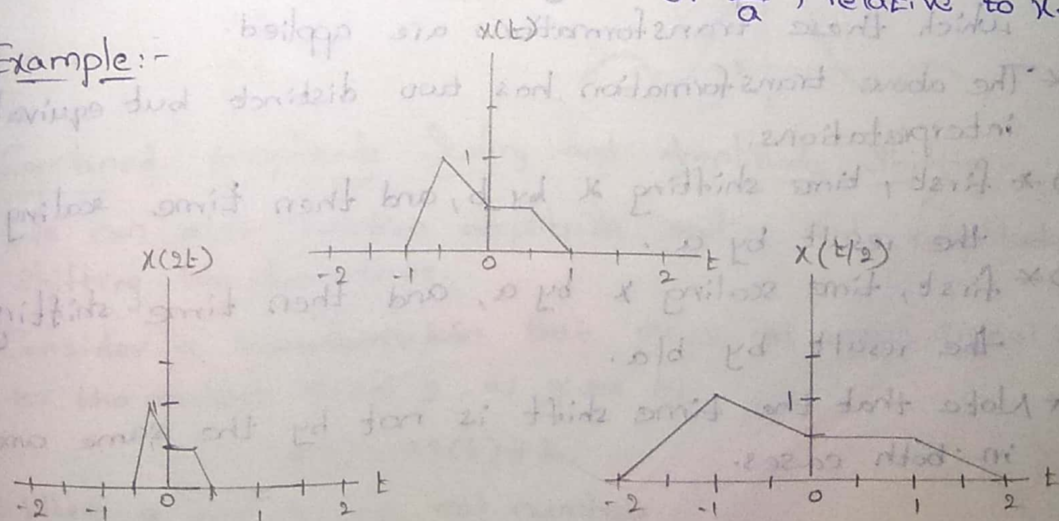
* Time compression/expansion (also called dilation) maps the input signal x to the output signal y as given by

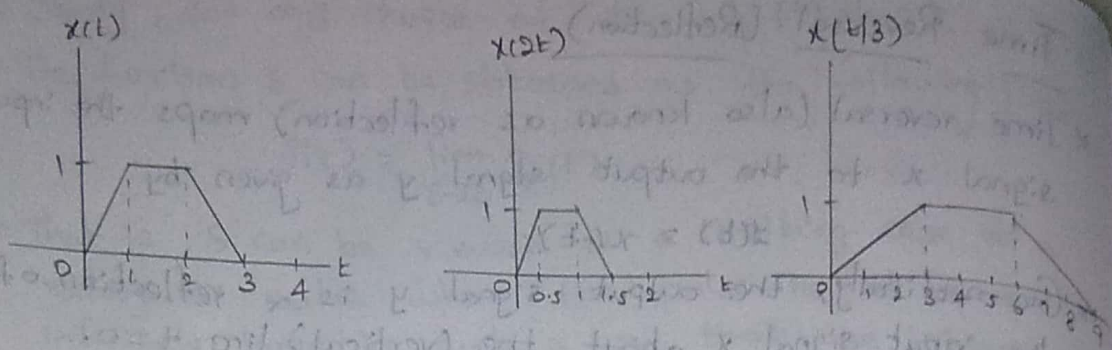
$$y(t) = x(at),$$

Where a is a strictly positive real number.

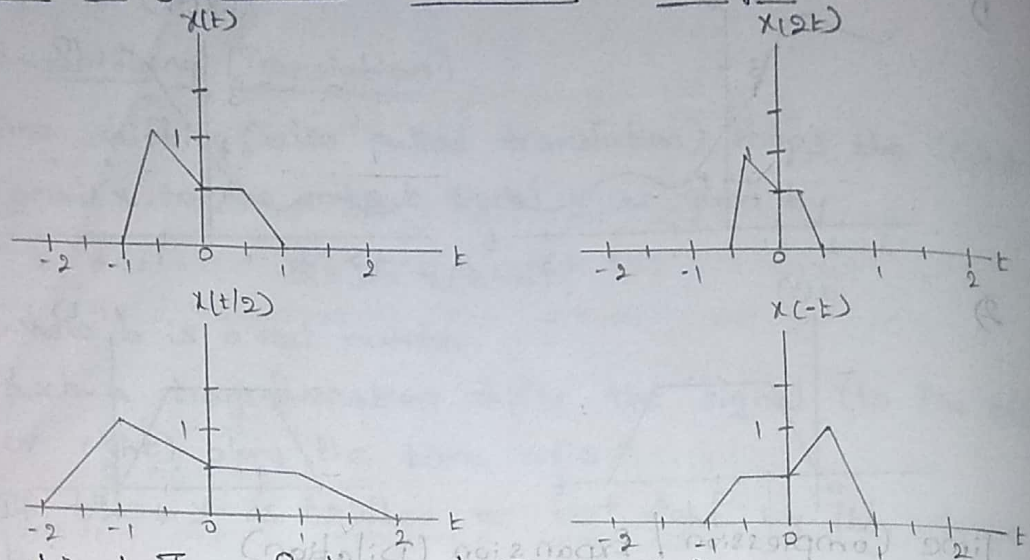
* Such a transformation is associated with a compression/expansion along the time axis.
* If $a>1$, y is compressed along the horizontal axis by a factor of a, relative to x.
* If $a<1$, y is expanded (i.e., stretched) along the horizontal axis by a factor of $\frac{1}{a}$, relative to x.

Example:-





Time Scaling (Dilation / Reflection): Example :-



Combined Time Scaling and Time Shifting :-

* Consider a transformation that maps the input signal x to the output signal y as given by

$$y(t) = x(at - b),$$

where a and b are real numbers and $a \neq 0$.

* The above transformation can be shown to be the combination of a time-scaling operation and time-shifting operation.
* Since time scaling and time shifting do not commute, we must be particularly careful about the order in which these transformations are applied.
* The above transformation has two distinct but equivalent interpretations:

→ * first, time shifting x by b, and then time scaling the result by a.

→ * first, time scaling x by a, and then time shifting the result by b/a.

* Note that the time shift is not by the same amount in both cases.

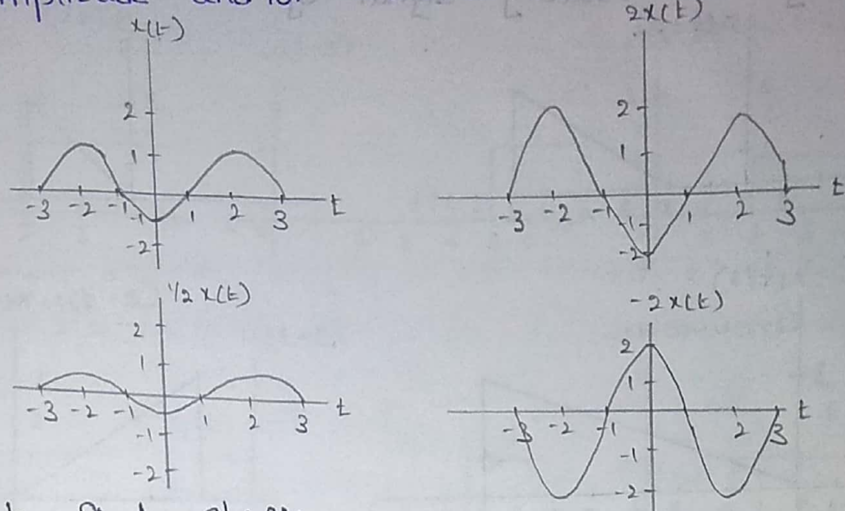
Amplitude Scaling :-

* Amplitude scaling maps the input signal x to the output signal y as given by

$$y(t) = a\,x(t).$$

Where a is a real number.

* Geometrically, the output signal y is expanded / compressed in amplitude and / or reflected about the horizontal axis.



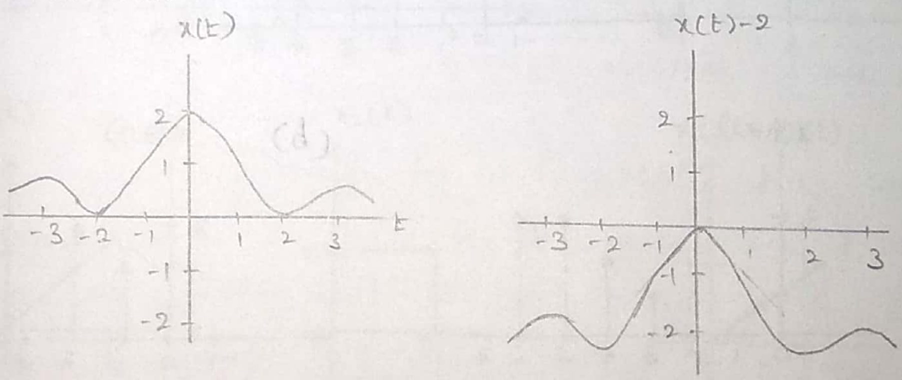
Amplitude ~~Scaly~~ Shifting :-

* Amplitude shifting maps the ~~output~~ input signal x to the output signal y as given by

$$y(t) = x(t) + b,$$

Where b is a real number.

* Geometrically, amplitude shifting adds a vertical displacement to x.



Combined Amplitude Scaling and Amplitude Shifting

* We can also combine amplitude scaling and amplitude shifting transformations.

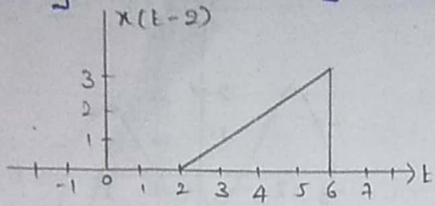
* Consider a transformation that maps the input signal x to the output signal y, as given by.

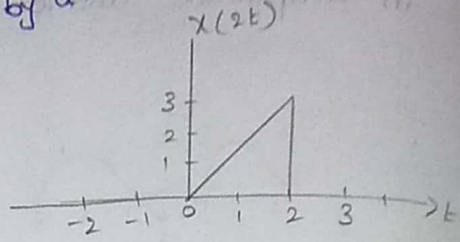
$$y(t) = a\,x(t) + b,$$

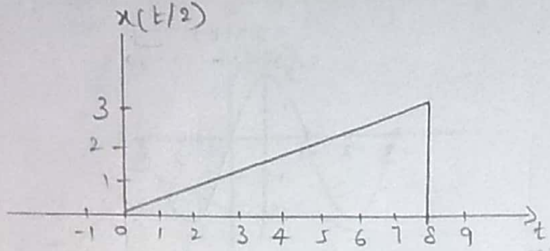
Where a and b are real numbers.

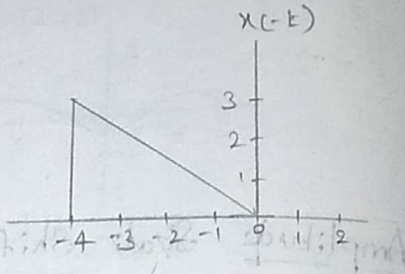
* Equivalently, the above transformation can be expressed as
$$y(t) = a \Sigma x(t) + {}_a b^{\Sigma}$$

* The above transformation is equivalent to:
→ first amplitude scaling x by a, and then amplitude shifting the resulting signal by b; or
→ first amplitude shifting x by b/a, and then amplitude scaling the resulting signal by a.

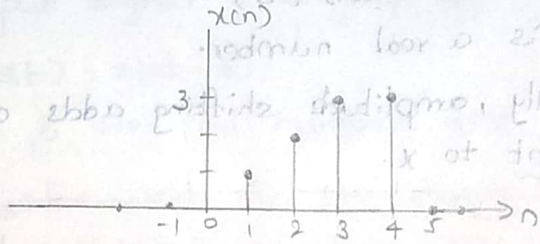




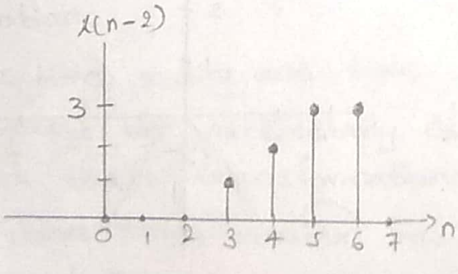




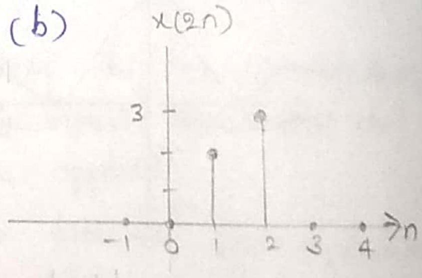
→ A discrete-time signal $x[n]$ is shown below. Sketch and label each of the following signals.
(a) $x[n-2]$, (b) $x[2n]$; (c) $x[-n]$, (d) $x[-n+2]$



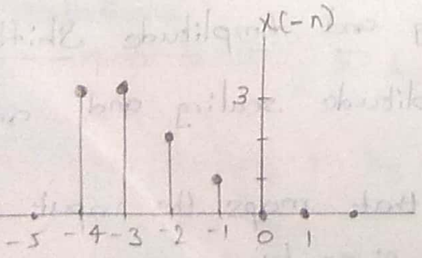
(a)



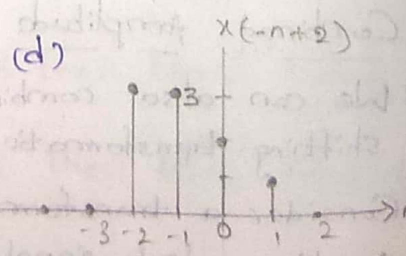
(b)



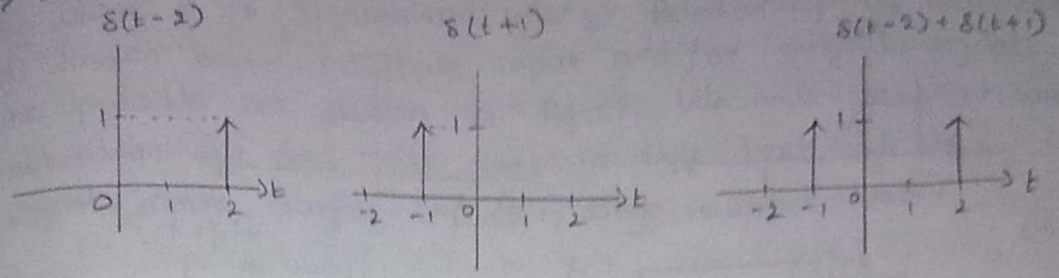
(c)



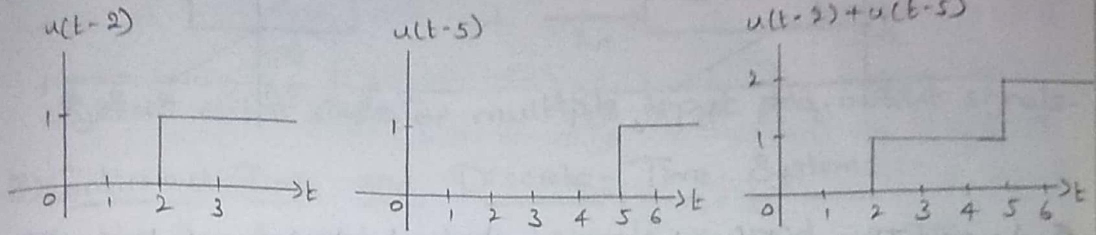
(d)



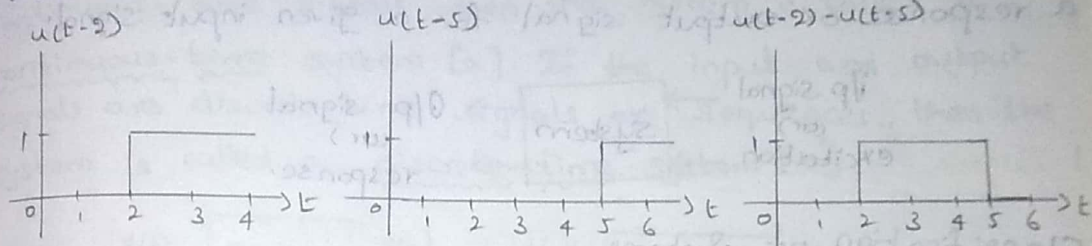
→ $\delta(t-2) + \delta(t+1)$



→ $u(t-2) + u(t-5)$

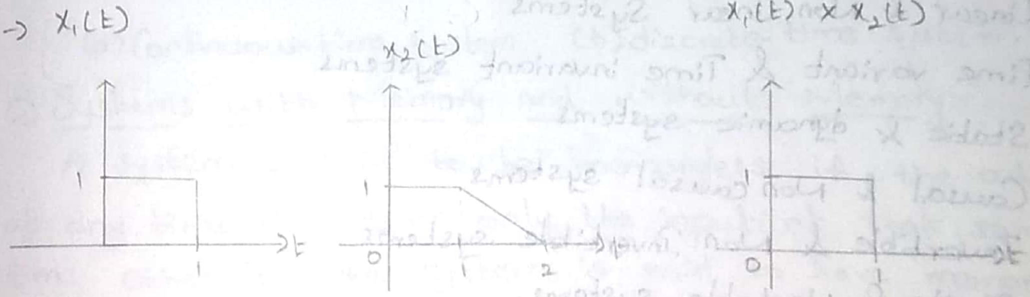


→ $u(t-2) - u(t-5)$

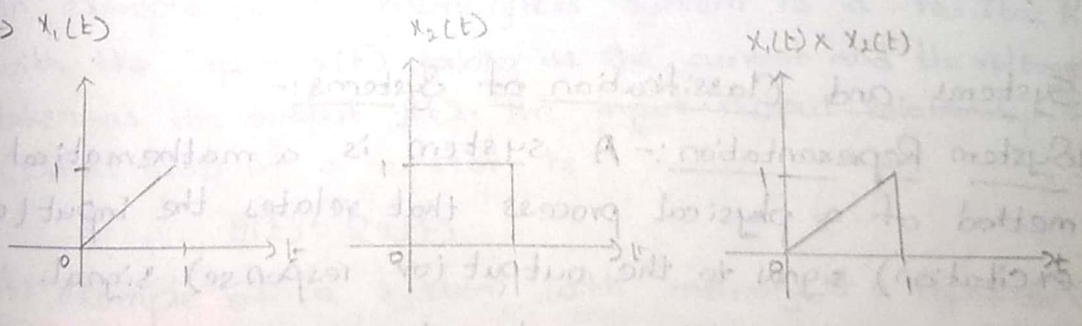


Signal Multiplication :-

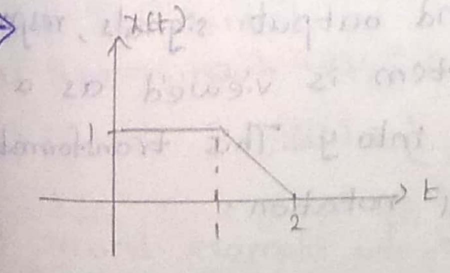
→ $x_1(t)$



→ $x_1(t)$

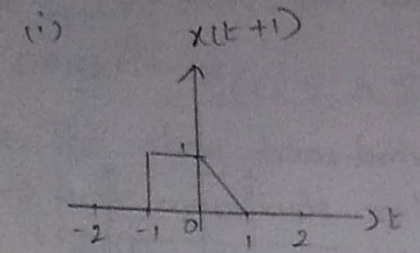


→ $x(t)$

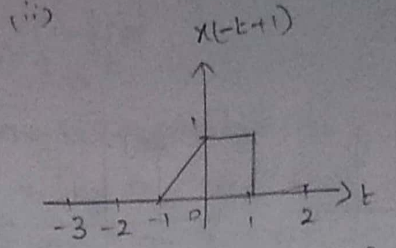


(i) $x(t+1)$, (ii) $x(-t+1)$, (iii) $x\left(\frac{3}{2}t\right)$, (iv) $x\left(\frac{3}{2}t+1\right)$

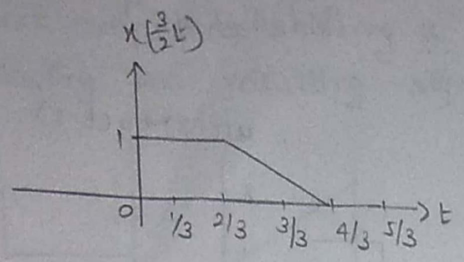
(i)

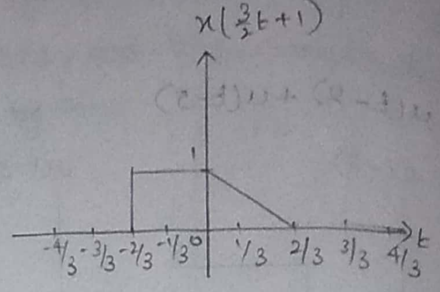


(ii)

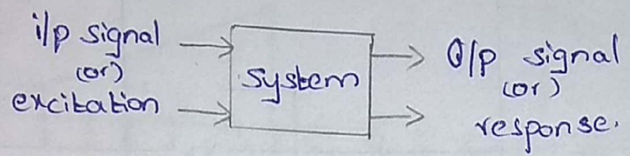


(iii)





System:- It is defined as a physical device that generate a response or output signal for a given input signal.



Classification of Systems

1.) Continuous & discrete time systems
2.) Linear & Non Linear Systems
3.) Time variant & Time invariant systems
4.) Static & dynamic systems
5.) Causal & Non causal systems
6.) Invertible & Non Invertible systems
7.) Stable & Unstable systems.

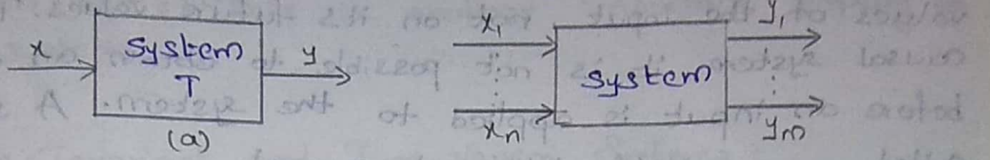
Systems and Classification of Systems:-

A) **System Representation**:- A system is a mathematical method of a physical process that relates the input (or excitation) signal to the output (or response) signal.

Let x and y be the input and output signals, respectively, of a system. Then the system is viewed as a transformation (or mapping) of x into y. This transformation is represented by the mathematical notation.

$$y = Tx.$$

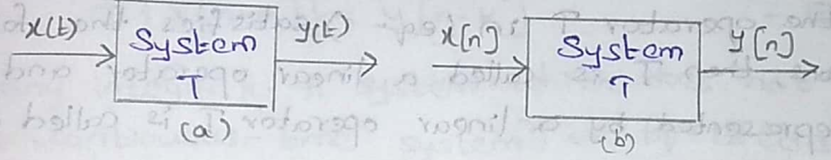
Where T is a operator representing some well-defined rule by which x is transformed into y. Relationship is depicted as shown below fig(1). Multiple input and / or output signals are possible as shown in fig. (2). We will restrict our attention for the most part in this text to the ~~signal-input~~ single-input, single-output case.



System with single or multiple input and output signals.

B) Continuous-Time and Discrete-Time Systems :-

If the input and output signals x and y are continuous-time signals, then the system is called a continuous-time system [a]. If the input and output signals are discrete-time signals or sequences, then the system is called a discrete-time system (b).



(a) Continuous-time system (b) discrete-time system.

C) Systems with Memory and without Memory:-

A system is said to be memoryless if the output at any time depends on only the input at that same time. Otherwise, the system is said to have memory. An example of a memoryless system is a resistor R with the input x(t) taken as the current and the voltage taken as the output y(t). The input-output relationship (Ohm's law) of a resistor is

$$y(t) = R\,x(t) \qquad -2$$

An example of a system with memory is a Capacitor C with the current as the input x(t) and the voltage as the output y(t); then

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)\,d\tau \qquad -3$$

A second example of a system with memory is a discrete-time system whose input and output sequences are related by

$$y[n] = \sum_{k=-\infty}^{n} x[k] \qquad -4$$

D.) Causal and Non causal Systems :-

A system is called causal if its output $y(t)$ at an arbitrary time $t = t_0$ depends on only the input $x(t)$ for $t \le t_0$. That is, the output of a causal system at the present time depends on only the present and/or past values of the input, not on its future values. Thus, in a causal system, it is not possible to obtain an output before an input is applied to the system. A system is called noncausal if it is not causal.

Examples of noncausal systems are

$$y(t) = x(t+1) \quad - 5$$

$$y(n) = x(-n). \quad - 6$$

** Note that all memoryless systems are causal, but not vice versa.

E.) Linear Systems and Non linear Systems :-

If the operator T in (eq - 1) satisfies the following two conditions, then T is called a linear operator and the system represented by a linear operator T is called a linear system.

1.) Additivity : Given that $Tx_1 = y_1$ and $Tx_2 = y_2$, then

$$T[x_1 + x_2] = y_1 + y_2 \quad - 7$$

for any signals x_1 and x_2.

2) Homogeneity (or Scaling) :-

$$T\{\alpha x\} = \alpha y \quad - 8$$

for any signals x and any scalar α.

Any system that does not satisfy (eq - 7) and/or (Eq - 8) is classified as a nonlinear system. Equations (6 & 7) can be combined into a single condition as

$$T\{\alpha_1 x_1 + \alpha_2 x_2\} = \alpha_1 y_1 + \alpha_2 y_2 \quad - 9$$

where α_1 and α_2 are arbitrary scalars. Equation (9) is known as the superposition property. Examples of linear systems are the resistor (eq. 2) and the capacitor (Eq. 3)]. Examples of nonlinear systems are

$$y = x^2 \qquad -10$$

$$y = \cos x \qquad -11$$

** Note that a consequence of the homogeneity (or scaling) property (eq-8) of linear systems is that a zero input yields a zero output. This follows readily by setting $\alpha = 0$ in (Eq-8). This is another important property of linear systems.

F.) Time-Invariant and Time-Varying Systems:

A system is called time-invariant if a time shift (delay or advance) in the input signal causes the same time shift in the output signal. Thus, for a continuous-time system, the system is time-invariant if

$$T\{x(t-\tau)\} = y(t-\tau) \qquad -12$$

for any real value of τ. for a discrete-time system, the system is time-invariant (or shift-invariant) if

$$T\{x[n-k]\} = y[n-k] \qquad -13$$

for any integer k. A system which does not satisfy eq-12 (continuous-time system) or Eq-13 (discrete-time system) is called a time-varying system. To check a system for time-invariance, we can compare the shifted output with the output produced by the shifted input.

G.) Linear Time-Invariant Systems

If the system is linear and also time-invariant, then it is called a linear time-invariant (LTI) system.

H.) Stable System:-

A system is bounded input / bounded output (BIBO) stable if for any bounded input x defined by

$$|x| \leq K_1$$

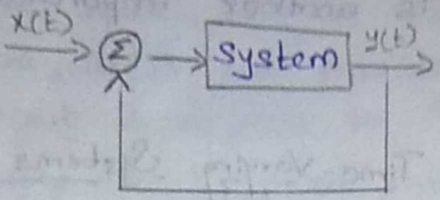
The corresponding output y is also bounded defined by

$$|y| \leq K_2$$

Where k_1 and k_2 are finite real constants. Note that there are many other definitions of stability.

I. Feedback Systems:

A special class of systems of great importance consists of system having feedback. In a feedback system, the output signal is fed back and added to the input to the system as shown below.



Problems :- Static & Dynamic System

1) $y(t) = x(2t)$

$t=0, \quad y(0) = x(0) \rightarrow$ present i/p
$t=1, \quad y(1) = x(2) \rightarrow$ future i/p
$t=-1, \quad y(-1) = x(-2) \rightarrow$ past i/p

Given system is dynamic.

2) $y(t) = x(-t)$

$t=0, \quad y(0) = x(0) \rightarrow$ present
$t=1, \quad y(1) = x(-1) \rightarrow$ past
$t=-1, \quad y(-1) = x(1) \rightarrow$ future

Dynamic system.

3) $y(t) = x(\sin t)$

$t=0, \quad y(0) = x(\sin 0) = x(0) \rightarrow$ present
$t=\pi, \quad y(\pi) = x(\sin\pi) = x(0)$
$y(3.14) = x(0) \rightarrow$ past

Dynamic system

4) $y(t) = e^{-2x(t)}$

$t=0, \quad y(0) = e^{-2x(0)}$
$t=1, \quad y(1) = e^{-2x(1)}$
$t=-1, \quad y(-1) = e^{-2x(-1)}$

Static system.

Causal & Non Causal system

1) $y(t) = x(t) + x(t-1)$

$t=0, \quad y(0) = \underbrace{x(0)}_{pre} + \underbrace{x(-1)}_{past}$

$t=1, \quad y(1) = \underbrace{x(1)}_{pre} + \underbrace{x(0)}_{past}$

$t=-1, \quad y(-1) = \underbrace{x(-1)}_{pre} + \underbrace{x(-2)}_{past}$

Causal system.

2.) $y(t) = x(2t)$

$t=0,\ y(0) = x(0) \rightarrow$ present

$t=1,\ y(1) = x(2) \rightarrow$ future — Noncausal System.

$t=-1,\ y(-1) = x(-2) \rightarrow$ past

3.) $y(t) = x\,e^{t}$

$t=0,\ y(0) = x(e^{0})$

$y(0) = x(1) \rightarrow$ future

$t=1,\ x(e^{1})\ y(1) = x(e^{1})$ — Noncausal system.

$t=-1,\ y(-1) = x(e^{-1})$

4.) $y(t) = x\sin t$

$t=0,\ y(0) = x(\sin 0) = x(0) \rightarrow$ present

$t=\pi/2,\ y(\pi/2) = x(\sin \pi/2)$

$y(1.57) = x(1) \rightarrow$ past — Noncausal system.

$t=-\pi/2,\ y(-\pi/2) = x(\sin(-\pi/2))$

$y(-1.57) = x(-1) \rightarrow$ future

5.) $y(t) = x(t/4)$

$t=0,\ y(0) = x(0) \rightarrow$ present

$t=1,\ y(1) = x(1/4) \rightarrow$ past — Noncausal system.

$t=-1,\ y(-1) = x(-1/4) \rightarrow$ future

6.) $y(t) = e^{2t}\cdot x(t-1)$

$t=0,\ y(0) = e^{2(0)}x(0-1)$

$y(0) = x(-1) \rightarrow$ past

$t=1,\ y(1) = e^{2(1)}\cdot x(1-1)$

$y(1) = e^{2(1)}x(0) \rightarrow$ past — Causal system.

$t=-1,\ y(-1) = e^{2(-1)}x(-1-1)$

$y(-1) = e^{2(-1)}x(-2) \rightarrow$ past

7.) $y(t) = x(t-1) \rightarrow$ Memory, Causal

8.) $y(t) = x(t+1) \rightarrow$ Memory, Noncausal

9.) $y(t) = x(t) \rightarrow$ Memoryless

10.) $y(t) = x(t) + x(t-1) \rightarrow$ Memory, causal

11.) $y(t) = x(t) + x(t+1) \rightarrow$ Memory, Non causal.

12.) $y(t+1) = x(t+1) \rightarrow$ Memoryless (or) static.

Time variant & time invariant system

Condition for time invariance is $\boxed{y(n,k) = y(n-k)}$

where $y(n,k) = T[x(n-k)]$ $\quad y(t,k) = y(t-k)$

Ex:- 1) $y(n) = x(n) + x(n-2)$

$$y(n,k) = T[x(n-k)] = x(n-k) + x(n-2-k)$$

$$y(n-k) = x(n-k) + x(n-k-2)$$

$$y(n,k) = y(n-k)$$

Time invariant system.

2) $y(n) = x(n) + nx(n-3)$

$$y(n,k) = T[x(n-k)] = x(n-k) + nx(n-k-3)$$

$$y(n-k) = x(n-k) + (n-k)x(n-k-3)$$

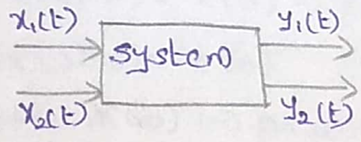
$$y(n,k) \neq y(n-k)$$

Time variant system.

Linear & Nonlinear system

* A system is said to be linear if it satisfies the superposition principle. Consider a system with input $x_1(t)$, $x_2(t)$ & output $y_1(t)$ & $y_2(t)$.

$$T[a_1x_1(t) + a_2x_2(t)] = a_1T[x_1(t)] + a_2T[x_2(t)]$$



$y(t) = x^2(t)$ $\quad T[x_1(t)] = x_1^2(t)$ $\quad$ Individual response $a_1x_1^2(t) + a_2x_2^2(t)$

↓

Nonlinear system $\quad T[x_2(t)] = x_2^2(t)$ $\quad \neq$

$$T[a_1x_1(t) + a_2x_2(t)] = [a_1x_1(t) + a_2x_2(t)]^2$$

Common response

$y(t) = x(t)$ $\quad T[x_1(t)] = x_1(t)$

↓ $\quad T[x_2(t)] = x_2(t)$ $\quad a_1x_1(t) + a_2x_2(t)$

Linear system

$$T[a_1x_1(t) + a_2x_2(t)] = a_1x_1(t) + a_2x_2(t)$$

Periodic Signals

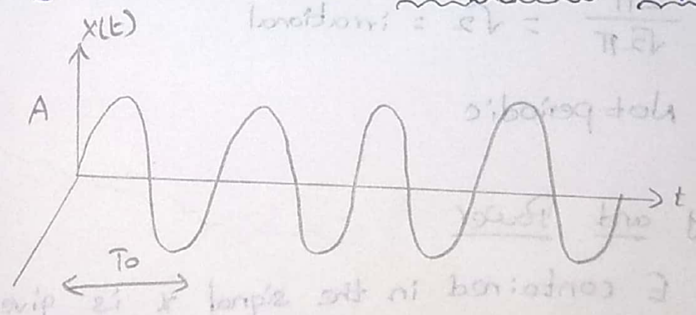
- A function x is said to be periodic with period T (or T-periodic) if, for some strictly-positive real constant T, the following condition holds:

$$x(t) = x(t+T) \text{ for all } t.$$

- A T-periodic function x is said to have frequency $\frac{1}{T}$ and angular frequency $\frac{2\pi}{T}$.
- A sequence x is said to be periodic with period N (or N-periodic) if, for some strictly-positive integer constant N, the following condition holds:

$$x(n) = x(n+N) \text{ for all } n.$$

- An N-periodic sequence x is said to have frequency $\frac{1}{N}$ and angular frequency $\frac{2\pi}{N}$.
- A function/sequence that is not periodic is said to be aperiodic.
- The period of a periodic signal is not unique. That is, a signal that is periodic with period T is also periodic with period KT, for every (strictly) positive integer K.
- The smallest period with which a signal is periodic is called the fundamental period and its corresponding frequency is called the fundamental frequency.



The above signal will repeat for every time interval T_0 hence it is periodic with period T_0.

(i) $x(t) = \cos\left(t + \frac{\pi}{4}\right)$

$$2\pi f t = 1$$

$$f = \frac{1}{2\pi}$$

$$T = \frac{1}{f} = \frac{1}{1/2\pi}$$

$$T = 2\pi$$

(ii) $x(t) = \sin\left(\frac{2\pi}{3}t\right)$

$$2\pi F = \frac{2\pi}{3}$$

$$F = \frac{1}{3}$$

$$T = \frac{1}{F} = 3$$

(v) $x(t) = \sin^2 t$

$$x(t) = \frac{1-\cos 2t}{2}$$

$$2\pi f = 2$$

$$f = \frac{1}{\pi}$$

$$T = \pi.$$

(iii) $x(t) = \cos\left(\frac{\pi}{3}t\right) + \sin\left(\frac{\pi}{4}t\right)$

$\cos(2\pi F t + \phi)$ $\quad$ $\sin(2\pi F t + \phi)$

$2\pi F = \pi/3$ $\quad$ $2\pi F = \frac{\pi}{4}$

$F = 1/6$ $\quad$ $F = \frac{1}{8}$

$T_1 = 6$ $\quad$ $T_2 = 8$

$$\frac{T_1}{T_2} = \frac{6}{8} = \frac{3}{4} = \text{rational number.}$$

$$T \Rightarrow 4T_1 = 3T_2$$

$$4 \cdot 6 + 3 \cdot 8 = 24$$

(iv) $x(t) = \cos t + \sin\sqrt{2}t.$

$2\pi F = 1$ $\quad$ $2\pi F = \sqrt{2}$

$F = \frac{1}{2\pi}$ $\quad$ $F = \frac{\sqrt{2}}{2\pi}$

$T_1 = 2\pi$ $\quad$ $T_2 = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$

$$\frac{T_1}{T_2} = \frac{2\pi}{\sqrt{2}\pi} = \sqrt{2} = \text{irrational}$$

Not periodic

Signal Energy and Power

* The energy E contained in the signal x is given by

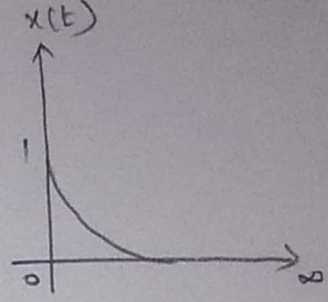
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt.$$

* A signal with finite energy is said to be an energy signal.
* The average power P contained in the signal x is given by

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt.$$

* A signal with (non zero) finite average power is said to be a power signal.

(i) $x(t) = e^{-at} \cdot u(t)$, $a > 0$



$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt.$$

$$= \int_{-\infty}^{\infty} \left[e^{-at} u(t) \right]^2 dt$$

$$= \int_{-\infty}^{0} e^{-at} \cdot 0 \cdot dt + \int_{0}^{\infty} (e^{-at})^2 dt.$$

$$= \int_{0}^{\infty} e^{-2at} dt \Rightarrow \left[\frac{e^{-2at}}{-2a} \right]_0^{\infty} \Rightarrow \left[-\frac{1}{2a} e^{-2at} \right]_0^{\infty}$$

$$= -\frac{1}{2a} \left[e^{-2a(\infty)} - e^{-2a(0)} \right]$$

$$= -\frac{1}{2a} [0 - 1] = -\frac{1}{-2a}$$

$$E = \frac{1}{2a} \neq \infty \qquad P = 0.$$